QuestIQ Academy

JEE Advanced 2010 – Paper 2 – Mathematics – Worked Solutions

Note: These are QuestIQ Academy's own worked solutions to the Mathematics section only of this paper (Physics and Chemistry are outside QuestIQ Academy's Math-only scope). Section 1 = single correct; Section 2 = single-digit integer; Section 3 = paragraph-based single correct; Section 4 = matrix-match.

Q.1

A_r, B_r, C_r = coefficient of x^r in $(1+x)^{10}, (1+x)^{20}, (1+x)^{30}$. Find $\sum_{r=1}^{10} A_r(B_{10}B_r - C_{10}A_r)$.

By Vandermonde's identity, $\sum_{r=0}^{10} A_rB_r = C_{10}$ and $\sum_{r=0}^{10} A_r^2 = B_{10}$, so the full sum ($r=0$ to 10) of $B_{10}A_rB_r - C_{10}A_r^2$ is $B_{10}C_{10} - C_{10}B_{10} = 0$. Subtracting the $r=0$ term ($B_{10} - C_{10}$) gives $C_{10} - B_{10}$.

Answer: (D) $C_{10} - B_{10}$

Q.2

$S = \{1, 2, 3, 4\}$. Total number of unordered pairs of disjoint subsets.

Each element goes to subset A, subset B, or neither: $3^4 = 81$ ordered pairs (A, B) with $A \cap B = \emptyset$. Only $(\emptyset, \emptyset)$ has $A = B$; the remaining 80 ordered pairs form 40 unordered pairs, plus the one self-paired case: $40 + 1 = 41$.

Answer: (D) 41

Q.3

f on $(-1, 1)$ with $e^{-x}f(x) = 2 + \int_0^x \sqrt{t^4+1}\,dt$. Find $(f^{-1})'(2)$.

At $x = 0$: $f(0) = 2$. Differentiating, $-e^{-x}f(x) + e^{-x}f'(x) = \sqrt{x^4+1}$; at $x = 0$: $-2 + f'(0) = 1 \Rightarrow f'(0) = 3$. So $(f^{-1})'(2) = 1/f'(0) = 1/3$.

Answer: (B) $1/3$

Q.4

Distance from $P(1, -2, 1)$ to plane $x + 2y - 2z = \alpha$ ($\alpha > 0$) is 5. Find the foot of the perpendicular.

$|-5 - \alpha|/3 = 5 \Rightarrow \alpha = 10$ (rejecting the negative solution). The foot is $F = P - \frac{P \cdot n - \alpha}{|n|^2} n = \left(\frac{8}{3}, \frac{4}{3}, -\frac{7}{3}\right)$.

Answer: (A) $\left(\frac{8}{3}, \frac{4}{3}, -\frac{7}{3}\right)$

Q.5

Parallelogram $ABCD$, $\vec{AB} = 2\hat{i} + 10\hat{j} + 11\hat{k}$, $\vec{AD} = -\hat{i} + 2\hat{j} + 2\hat{k}$. AD rotated by acute α to become $\perp AB$. Find $\cos\alpha$.

$|\vec{AD}| = 3, |\vec{AB}| = 15$, and $\cos\beta = \vec{AD} \cdot \vec{AB}/45 = 40/45 = 8/9$ where β is the angle between them. The rotation angle is $\alpha = 90^\circ - \beta$, so $\cos\alpha = \sin\beta = \sqrt{1 - 64/81} = \sqrt{17}/9$.

Answer: (B) $\sqrt{17}/9$

Q.6

Signal green (prob. 4/5) or red (1/5) sent A→B, each station correct with prob. 3/4. Given B receives green, find P(original green).

$P(\text{green at B} \mid \text{green sent}) = \left(\frac{3}{4}\right)^2 + \left(\frac{1}{4}\right)^2 = \frac{5}{8}$; $P(\text{green at B} \mid \text{red sent}) = 2 \cdot \frac{3}{4} \cdot \frac{1}{4} = \frac{3}{8}$. Bayes: $\dfrac{\frac{4}{5} \cdot \frac{5}{8}}{\frac{4}{5} \cdot \frac{5}{8} + \frac{1}{5} \cdot \frac{3}{8}} = \dfrac{20/40}{23/40} = \dfrac{20}{23}$.

Answer: (C) 20/23

Q.7

Two parallel chords of a circle radius 2, distance $\sqrt{3} + 1$ apart, subtend angles $\pi/k, 2\pi/k$ at the centre ($k > 0$). Find $[k]$.

Chord distances from centre are $2\cos(\pi/(2k))$ and $2\cos(\pi/k)$; since their sum can't exceed 2 on the same side, they lie on opposite sides: $2\cos\frac{\pi}{2k} + 2\cos\frac{\pi}{k} = \sqrt{3} + 1$. Testing $k = 3$ gives $2\cos 30^\circ + 2\cos 60^\circ = \sqrt{3} + 1$ exactly.

Answer: 3

Q.8

$\triangle ABC$, $a = 6, b = 10$, area $= 15\sqrt{3}$, $\angle ACB$ obtuse. Find r^2.

$\sin C = \frac{2 \cdot 15\sqrt{3}}{60} = \frac{\sqrt{3}}{2}$, and obtuseness forces $C = 120^\circ$. Then $c^2 = 36 + 100 + 60 = 196 \Rightarrow c = 14$, $s = 15$, and $r = \text{Area}/s = \sqrt{3}$.

Answer: 3

Q.9

$f'(x) = 2010(x - 2009)(x - 2010)^2(x - 2011)^3(x - 2012)^4$, $f = \ln g$, $g > 0$ on $\mathbb{R}$. Number of local maxima of g.

Since $g' = g \cdot f'$ and $g > 0$, the sign of g' matches f', which (ignoring the always-nonnegative even-power factors) is governed by $(x - 2009)(x - 2011)$: positive, then negative, then positive. This gives one local max (at $x = 2009$) and one local min (at $x = 2011$); $x = 2010, 2012$ are not extrema (sign doesn't change there).

Answer: 1

Q.10

$a_1 = 15$, $27 - 2a_2 > 0$, $a_k = 2a_{k-1} - a_{k-2}$ for $k = 3, \ldots, 11$ (an AP). If $\frac{1}{11}\sum a_k^2 = 90$, find $\frac{1}{11}\sum a_k$.

With common difference d: $\sum_{k=1}^{11} a_k^2 = 2475 + 1650d + 385d^2 = 990$, giving $7d^2 + 30d + 27 = 0 \Rightarrow d = -3$ or $-9/7$; only $d = -3$ satisfies $27 - 2a_2 > 0$ (i.e. $d < -1.5$). The average equals the median term $a_6 = a_1 + 5d = 0$.

Answer: 0

Q.11

A, B as given (B skew-symmetric 3×3, hence $\det B = 0$); $\det(\operatorname{adj} A) + \det(\operatorname{adj} B) = 10^6$. Find $[k]$.

Cofactor expansion gives $\det A = (2k + 1)^3$. Using $\det(\operatorname{adj} A) = (\det A)^2$: $(2k + 1)^6 = 10^6 \Rightarrow 2k + 1 = 10 \Rightarrow k = 4.5$.

Answer: 4

Q.12

[Paragraph] $f(x) = 1 + 2x + 3x^2 + 4x^3$; s = sum of distinct real roots. Interval containing s.

$f'(x) = 12x^2 + 6x + 2$ has negative discriminant, so f is strictly increasing – exactly one real root, found numerically near -0.605.

Answer: (C) $(-\frac{3}{4}, -\frac{1}{2})$

Q.13

[Paragraph, same f] With $t = |s|$, the area bounded by $y = f(x)$, $x = 0, y = 0, x = t$ lies in which interval?

Area $= \int_0^t f(x)dx = t^4 + t^3 + t^2 + t \approx 1.33$ for $t \approx 0.605$.

Answer: (A) $(\frac{3}{4}, 3)$

Q.14

[Paragraph, same f] Behaviour of $f'(x)$ on $(-t, t)$.

$f'(x) = 12x^2 + 6x + 2$ is an upward parabola with vertex at $x = -1/4$: decreasing before, increasing after.

Answer: (B) decreasing on $(-t, -\frac{1}{4})$**, increasing on** $(-\frac{1}{4}, t)$

Q.15

[Paragraph] Tangents from $P(3, 4)$ to $\frac{x^2}{9} + \frac{y^2}{4} = 1$ touch at A, B. Find A, B.

Chord of contact: $x + 3y = 3$. Substituting into the ellipse gives $y = 0$ (point $(3, 0)$) or $y = 8/5$ (point $(-9/5, 8/5)$).

Answer: (D) $(3, 0)$ **and** $(-9/5, 8/5)$

Q.16

[Paragraph, same points] Orthocentre of $\triangle PAB$.

Since PA is vertical ($x = 3$), the altitude from B is horizontal: $y = 8/5$. PB has slope $1/2$, so the altitude from A has slope -2: $y = -2x + 6$. Intersection: $(11/5, 8/5)$.
Answer: (C) $(11/5, 8/5)$

Q.17

[Paragraph, same points] Locus of points equidistant from $P(3, 4)$ and line AB.

Line AB is $x+3y = 3$ (the chord of contact). Equating $\sqrt{(x-3)^2 + (y-4)^2}$ to the point-to-line distance and simplifying gives $9x^2 + y^2 - 6xy - 54x - 62y + 241 = 0$.
Answer: (A) $9x^2 + y^2 - 6xy - 54x - 62y + 241 = 0$

Q.18

[Matrix match] Match Column I to Column II (p=ellipse ecc. $4/5$, q: $\operatorname{Im} z = 0$, r: $|\operatorname{Im} z| \le 1$, s: $|\operatorname{Re} z| \le 2$, t: $|z| \le 3$).

(A) $|z - i|z|| = |z + i|z||$ forces z onto the real axis exactly ($= q$), automatically satisfying r. (B) $|z + 4| + |z - 4| = 10$ is an ellipse with $a = 5, c = 4, e = 4/5$ – exactly p. (C) $|w| = 2, z = w - 1/w$ traces an ellipse with semi-axes $3/2, 5/2$ (eccentricity also $4/5$), contained in $|\operatorname{Re} z| \le 3/2 \subset s$ and $|z| \le 5/2 \subset t$. (D) $|w| = 1, z = w + 1/w = 2 \operatorname{Re} w \in [-2, 2]$ is real-valued and bounded, contained in q, r, s, t all at once.
Answer: A→q,r; B→p; C→p,s,t; D→q,r,s,t