QuestIQ Academy

JEE Advanced 2018 – Paper 1 – Mathematics – Worked Solutions

Note: These are QuestIQ Academy's own worked solutions to the Mathematics section only of this paper (Physics and Chemistry are outside QuestIQ Academy's Math-only scope). Section 1 = one or more correct; Section 2 = numerical value; Section 3 = single correct (paragraph-based).

Q.1

With $\arg(z) \in (-\pi, \pi]$*, which statement(s) is/are FALSE?*

(A) $-1-i$ lies in the third quadrant, so its principal argument is $-3\pi/4$, not $\pi/4$ – FALSE (correctly identified). (B) As $t \to 0^-$, $\arg(-1+it) \to -\pi$, but $f(0) = \arg(-1) = \pi$; the left- and right-hand limits disagree with the defined value, so f is discontinuous at $t = 0$ – the claim of continuity is FALSE. (C) $\arg(z_1/z_2) - \arg z_1 + \arg z_2$ is always an integer multiple of 2π – this is a TRUE identity, so it is not among the false statements. (D) The locus described is in general a circular arc through z_1, z_3 (only a line if z_1, z_2, z_3 happen to be collinear) – the blanket claim of a straight line is FALSE.

Answer: (A), (B), (D)

Q.2

$\triangle PQR$*:* $\angle PQR = 30°$*,* $PQ = 10\sqrt{3}$*,* $QR = 10$*.*

By the cosine rule, $PR^2 = 300 + 100 - 2(10\sqrt{3})(10)\cos 30° = 100$, so $PR = 10 = QR$ (isosceles). By the sine rule $\sin P = 1/2 \Rightarrow P = 30°$ (rejecting $150°$), so $R = 120°$ – (A)'s claim of $45°$ is false. Area $= \frac{1}{2}(10\sqrt{3})(10)\sin 30° = 25\sqrt{3}$ with $\angle QRP = 120°$ (B true). Semi-perimeter $s = 5\sqrt{3} + 10$, inradius $=$ Area$/s = 10\sqrt{3} - 15$ (C true). Circumradius from $QR/\sin P = 2R_c \Rightarrow R_c = 10$, circumcircle area $= 100\pi$ (D true).

Answer: (B), (C), (D)

Q.3

$P_1 : 2x + y - z = 3,\ P_2 : x + 2y + z = 2.$

Direction of intersection line $= n_1 \times n_2 = (3, -3, 3) \parallel (1, -1, 1)$, not $(1, 2, -1)$ – (A) false. The given line $\frac{3x-4}{9} = \frac{1-3y}{9} = \frac{z}{3}$ has direction $(1, -1, 1)$ too – it is parallel to, not perpendicular to, the intersection line – (B) false. $\cos\theta = \dfrac{|n_1 \cdot n_2|}{|n_1||n_2|} = \dfrac{3}{6} = \dfrac{1}{2} \Rightarrow \theta = 60°$ (C true). Plane P_3 through

$(4, 2, -2)$ with normal $(1, -1, 1)$ is $x - y + z = 0$; distance from $(2, 1, 1)$ is $\dfrac{|2-1+1|}{\sqrt{3}} = \dfrac{2}{\sqrt{3}}$ (D true).
Answer: (C), (D)

Q.4

$f : \mathbb{R} \to [-2, 2]$ twice differentiable, $(f(0))^2 + (f'(0))^2 = 85$.

Since $f(0)^2 \le 4$, $|f'(0)| \ge 9$, so f' is nonzero (hence sign-constant) near 0, making f one-one on some interval around 0 (A true). Since $f(0) - f(-4) \le 4$ (both values in $[-2, 2]$), by the Mean Value Theorem some $x_0 \in (-4, 0)$ has $|f'(x_0)| = |f(0) - f(-4)|/4 \le 1$ (B true). There is no general reason $\lim_{x\to\infty} f(x) = 1$ (C false – unjustified). Considering $g(x) = f(x)^2 + f'(x)^2$ with $g(0) = 85$ and g bounded (since f is bounded and, by an argument like (B), f' cannot stay large), g is non-monotonic, so $g'(x) = 2f'(x)[f(x) + f''(x)] = 0$ at some point with $f' \ne 0$, forcing $f + f'' = 0$ there (D true).
Answer: (A), (B), (D)

Q.5

$f'(x) = e^{f(x)-g(x)} g'(x)$, $f(1) = g(2) = 1$.

Rewriting: $\frac{d}{dx}\left[e^{-g(x)} - e^{-f(x)}\right] = 0$, so $e^{-g(x)} - e^{-f(x)} = k$ (constant) for all x. Evaluating at $x = 1$ and $x = 2$ and equating gives $e^{-g(1)} + e^{-f(2)} = 2/e$. Since both terms are strictly positive, each is strictly less than $2/e = e^{-(1-\ln 2)}$, so $f(2) > 1 - \ln 2$ and $g(1) > 1 - \ln 2$.
Answer: (B), (C)

Q.6

$f(x) = 1 - 2x + \int_0^x e^{x-t} f(t)\, dt$, $x \ge 0$.

Differentiating and simplifying using the original equation gives $f'(x) - 2f(x) = 2x - 3$, a linear ODE with $f(0) = 1$; solving gives $f(x) = 1 - x$ (verified directly by substitution back into the integral equation). So $f(1) = 0 \ne 2$ (A false) and $f(2) = -1$ (B true, passes through $(2, -1)$). The area between $y = 1 - x$ and $y = \sqrt{1 - x^2}$ on $[0, 1]$ is $\int_0^1 [\sqrt{1 - x^2} - (1 - x)] dx = \frac{\pi}{4} - \frac{1}{2} = \frac{\pi - 2}{4}$ (C true, D false).
Answer: (B), (C)

Q.7

Value of $\left((\log_2 9)^2\right)^{1/\log_2(\log_2 9)} \times (\sqrt{7})^{1/\log_4 7}$.

Using the identity $x^{k/\log_b x} = b^k$: the first factor equals $2^2 = 4$. For the second, taking logs directly: $\ln y = \frac{\ln 4}{\ln 7} \cdot \frac{1}{2} \ln 7 = \frac{1}{2} \ln 4 = \ln 2$, so $y = 2$. Product $= 4 \times 2$.
Answer: 8.00

Q.8

5-digit numbers divisible by 4, digits from $\{1, 2, 3, 4, 5\}$*, repetition allowed.*

The first three digits are free (5^3 ways); divisibility by 4 depends only on the last two digits. Checking all 5×5 endings shows exactly one valid last digit for each choice of the tens digit, giving 5 valid endings. Total $= 5^3 \times 5$.

Answer: 625.00

Q.9

X *= first 2018 terms of* $1, 6, 11, \ldots$*;* Y *= first 2018 terms of* $9, 16, 23, \ldots$*. Find* $|X \cup Y|$.

$X \cap Y$ consists of numbers $\equiv 1 \pmod 5$ and $\equiv 2 \pmod 7$, i.e. $\equiv 16 \pmod{35}$ by CRT. Counting terms of this form within both ranges gives 288 common elements. So $|X \cup Y| = 2018 + 2018 - 288$.

Answer: 3748.00

Q.10

$y_n = \frac{1}{n}\left[(n+1)(n+2)\cdots(n+n)\right]^{1/n}$*. Find* $[L]$ *where* $L = \lim y_n$.

$\ln y_n = \frac{1}{n}\sum_{k=1}^{n} \ln(1 + k/n) \to \int_0^1 \ln(1+x)\,dx = 2\ln 2 - 1$. So $L = e^{2\ln 2 - 1} = 4/e \approx 1.47$.

Answer: 1.00

Q.11

Unit vectors $\vec{a}, \vec{b}$ *with* $\vec{a} \cdot \vec{b} = 0$*;* $\vec{c} = x\vec{a} + y\vec{b} + (\vec{a} \times \vec{b})$, $|\vec{c}| = 2$*, equal angle to* $\vec{a}, \vec{b}$*. Find* $8\cos^2\alpha$.

$|\vec{c}|^2 = x^2 + y^2 + 1 = 4 \Rightarrow x^2 + y^2 = 3$. Equal angles force $x = y$ (since $\cos\alpha = x/2 = y/2$), giving $x = y = \pm\sqrt{3/2}$, so $\cos^2\alpha = x^2/4 = 3/8$.

Answer: 3.00

Q.12

$\sqrt{3}\,a\cos x + 2b\sin x = c$ *has distinct roots* $\alpha, \beta \in [-\pi/2, \pi/2]$ *with* $\alpha + \beta = \pi/3$*. Find* b/a.

Writing the LHS as $R\cos(x - \varphi)$, the two roots are symmetric about φ, so $2\varphi = \alpha + \beta = \pi/3 \Rightarrow \varphi = \pi/6$. Since $\tan\varphi = \dfrac{2b}{\sqrt{3}a} = \tan(\pi/6) = \dfrac{1}{\sqrt{3}}$, we get $2b = a$.

Answer: 0.50

Q.13

Farmer's land $\triangle P(0,0), Q(1,1), R(2,0)$*; region between* PQ *and* $y = x^n$ $(n > 1)$ *is* 30% *of the triangle's area.*

Area of $\triangle PQR = 1$. The region's area is $\int_0^1 (x - x^n)\,dx = \frac{1}{2} - \frac{1}{n+1}$. Setting this to 0.3 gives $\frac{1}{n+1} = 0.2 \Rightarrow n = 4$.

Answer: 4.00

Q.14

Circle $S : x^2 + y^2 = 4$; $P_0(1, 1)$. Chords E_1E_2 (horizontal), F_1F_2 (vertical), G_1G_2 (slope -1) through P_0; tangents at the endpoints of each chord meet at E_3, F_3, G_3. These lie on:

E_1E_2 is $y = 1$, meeting the circle at $(\pm\sqrt{3}, 1)$; the tangents there meet at $E_3 = (0, 4)$. F_1F_2 is $x = 1$, giving $F_3 = (4, 0)$. G_1G_2 is $x + y = 2$, meeting the circle at $(0, 2), (2, 0)$, whose tangents meet at $G_3 = (2, 2)$. All three points satisfy $x + y = 4$.

Answer: (A) $x + y = 4$

Q.15

P on $S : x^2 + y^2 = 4$ (both coordinates positive); tangent at P meets the axes at M, N. Locus of the midpoint of MN.

With $P = (2\cos\theta, 2\sin\theta)$, the tangent meets the axes at $M = (2\sec\theta, 0)$, $N = (0, 2\csc\theta)$, so the midpoint is $(\sec\theta, \csc\theta)$. Eliminating θ via $\cos^2\theta + \sin^2\theta = 1$ gives $\frac{1}{h^2} + \frac{1}{k^2} = 1$, i.e. $x^2 + y^2 = x^2y^2$.

Answer: (D) $x^2 + y^2 = x^2y^2$

Q.16

Probability S_1 gets seat R_1 and none of $S_2, \ldots, S_5$ gets their own original seat.

Fixing $S_1 \to R_1$, the remaining 4 students must be completely deranged: $D_4 = 9$ ways, out of $5! = 120$ total arrangements.

Answer: (A) 3/40

Q.17

T_i: students S_i, S_{i+1} $(i = 1, \ldots, 4)$ are NOT seated adjacently. Find $P(T_1 \cap T_2 \cap T_3 \cap T_4)$.

By inclusion–exclusion on the four "bad" adjacency events (treating adjacent pairs as blocks): $\sum |A_i| = 192$, $\sum |A_i \cap A_j| = 108$, $\sum |A_i \cap A_j \cap A_k| = 24$, $|A_1 \cap \cdots \cap A_4| = 2$, giving $|A_1 \cup \cdots \cup A_4| = 192 - 108 + 24 - 2 = 106$. Favourable count $= 120 - 106 = 14$, probability $= 14/120$.

Answer: (C) 7/60