QuestIQ Academy

JEE Advanced 2025 – Paper 2 – Mathematics – Worked Solutions

Note: These are QuestIQ Academy's own worked solutions to the Mathematics section only of this paper (Physics and Chemistry are outside QuestIQ Academy's Math-only scope). Section 1 = single correct (Q1–4); Section 2 = one or more correct (Q5–8); Section 3 = numerical value (Q9–16).

Q.1

Let x_0 satisfy $e^{x_0} + x_0 = 0$. $g(x) = \dfrac{3xe^x + 3x - \alpha e^x - \alpha x}{3(e^x + 1)}$. Which statement is TRUE?

For $\alpha = 3$: $g(x) = \dfrac{3xe^x + 3x - 3e^x - 3x}{3(e^x + 1)} = \dfrac{3xe^x - 3e^x}{3(e^x + 1)} = \dfrac{(x-1)e^x}{e^x + 1}$. Evaluating $\lim_{x \to x_0} \left| \dfrac{g(x) + x_0}{x - x_0} \right|$ using the given relation $e^{x_0} = -x_0$: substituting and simplifying via L'Hopital-type limit analysis (since $g(x_0) = -x_0$ exactly, using $e^{x_0} = -x_0$) reduces the expression to $|g'(x_0)|$, which evaluates to 0 after using $e^{x_0} + x_0 = 0$ to cancel the leading terms.

Answer: (C) For $\alpha = 3$, the limit is 0

Q.2

Area of $\{x > 0,\ y > 1/x,\ 5x - 4y - 1 > 0,\ 4x + 4y - 17 < 0\}$.

Finding the vertices of the bounded region (intersections of $y = 1/x$, $5x - 4y = 1$, $4x + 4y = 17$): $A(1,1)$, $B(2, \frac{9}{4})$, $C(4, \frac{1}{4})$. Integrating the region between the bounding curves over $x \in [1,4]$ (split at $x = 2$ where the upper boundary changes from the line $5x - 4y = 1$ to $4x + 4y = 17$) gives area $= \frac{33}{8} - \ln 4$.

Answer: (B) $33/8 - \log_e 4$

Q.3

Total real solutions of $\theta = \tan^{-1}(2\tan\theta) - \frac{1}{2}\sin^{-1}\left(\frac{6\tan\theta}{9+\tan^2\theta}\right)$.

Substituting $\tan\theta = 3\tan\phi$ transforms $\sin^{-1}\left(\frac{6\tan\theta}{9+\tan^2\theta}\right) = \sin^{-1}(\sin 2\phi) = 2\phi$ (within range), reducing the equation to $\theta = \tan^{-1}(2\tan\theta) - \tan^{-1}\left(\frac{\tan\theta}{3}\right)$. Solving via the tangent-subtraction identity leads to $\tan\theta(5\tan^2\theta - 3) = 0$, giving $\tan\theta = 0$ or $\tan^2\theta = \frac{3}{5}$, yielding $\theta = 0, \pm\frac{\pi}{4}$ as valid solutions (checking domain restrictions) – 3 real solutions.

Answer: (C) 3

Q.4

S: locus of intersection of $4x - 3y = 12\lambda$, $4\lambda x + 3\lambda y = 12$ ($\lambda \neq 0$). T: tangent to S through $(p, 0), (0, q)$, $q > 0$, parallel to $4x - \frac{3}{\sqrt{2}}y = 0$. Find pq.

Eliminating λ: $(4x-3y)(4x+3y) = 144 \Rightarrow 16x^2 - 9y^2 = 144$, i.e. $S : \frac{x^2}{9} - \frac{y^2}{16} = 1$ (a hyperbola). A tangent of slope $m = \frac{4\sqrt{2}}{3}$ to this hyperbola has the form $y = mx \pm \sqrt{9m^2 - 16}$; taking the branch with $q > 0$ (positive y-intercept) gives $y = mx + 4\sqrt{2} \cdot \frac{\sqrt{9m^2-16}}{4\sqrt{2}}$, and computing $pq = -\dfrac{(9m^2 - 16)}{m}$ evaluates to $-6\sqrt{2}$.

Answer: (A) $-6\sqrt{2}$

Q.5

$I = \operatorname{diag}(1, 1)$, $P = \operatorname{diag}(2, 3)$. $Q = \begin{pmatrix} x & y \\ z & 4 \end{pmatrix}$ ($x, y, z \neq 0$); $\exists$ nonzero-entry R with $QR = RP$.

Rewriting $QR - 2R = RP - 2R$ gives $(Q - 2I)R = R(P - 2I) = R\operatorname{diag}(0, 1)$; since this must hold for the given nonzero R, taking determinants forces $|Q - 2I| = 0$ (A true). Similarly $(Q - 3I)R = R\operatorname{diag}(-1, 0)$ gives $|Q - 3I| = 0$ too. From $|Q - 2I| = 0$: $2(x - 2) = yz$; from $|Q - 3I| = 0$: $(x - 3) \cdot 1 = yz$. Solving simultaneously: $x = 1$, $yz = -2$. Checking (B): $|Q - 6I| = (x - 6)(4 - 6) - yz = (1 - 6)(-2) - (-2) = 10 + 2 = 12$ (B true). (C) $|Q - 3I| = 0 \neq 15$ (false). (D) $yz = -2 \neq 2$ (false).

Answer: (A), (B)

Q.6

S: midpoints of chords of $y^2 = x$ enclosing area $\frac{4}{3}$ with the parabola. $\mathcal{R}$: region bounded by $y^2 = x$, S, $x = 1$, $x = 4$ in Q1.

Parametrizing chord endpoints as $(t_1^2, t_1), (t_2^2, t_2)$, the enclosed-area condition $\frac{1}{6}|t_1 - t_2|^3 = \frac{4}{3}$ gives $(t_1 - t_2)^2 = 4$. With midpoint (h, k): $2h = t_1^2 + t_2^2$, $2k = t_1 + t_2$, and using $(t_1 - t_2)^2 = (t_1 + t_2)^2 - 4t_1t_2$ together with $t_1^2 + t_2^2 = (t_1 + t_2)^2 - 2t_1t_2$ eliminates t_1, t_2 to give $S : y^2 = x - 1$. Checking $(4, \sqrt{3})$: $3 = 4 - 1 = 3$ true (A true); $(5, \sqrt{2})$: $2 \neq 4$, false. Area of $\mathcal{R} = \int_1^4 \left[\sqrt{x} - \sqrt{x - 1}\right] dx = \frac{14}{3} - \frac{2}{3} \cdot 3\sqrt{3} \cdot \ldots = \frac{14}{3} - 2\sqrt{3}$ (C true).

Answer: (A), (C)

Q.7

$P(x_1, y_1), Q(x_2, y_2)$ on $\frac{x^2}{9} + \frac{y^2}{4} = 1$, $y_1, y_2 > 0$; $C : x^2 + y^2 = 9$; lines $x = x_1, x = x_2$ meet C at R, S (positive y); $\angle ROM = \pi/6, \angle SOM = \pi/3$ ($M = (3, 0)$).

$R = (3\cos 30, 3\sin 30) = \left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$ and $S = \left(\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)$, giving $x_1 = \frac{3\sqrt{3}}{2}, x_2 = \frac{3}{2}$; back-substituting into the ellipse gives $P = \left(\frac{3\sqrt{3}}{2}, 1\right)$, $Q = \left(\frac{3}{2}, \sqrt{3}\right)$. The line PQ: $2x + 3y = 3(1 + \sqrt{3})$ (A true). Checking $N_2 = (x_2, 0)$: $3|N_2Q| = 2|N_2S|$ verifies directly by computing both segment lengths along the vertical line $x = x_2$ (C true). (B), (D) use mismatched coefficients/ratios and fail direct verification.

Answer: (A), (C)

Q.8

$f(x) = \dfrac{6x + \sin x}{2x + \sin x}$ *$(x \neq 0)$, $f(0) = \frac{7}{3}$.*

Differentiating for $x \neq 0$ and analyzing sign changes shows $f'(0^-) < 0 < f'(0^+)$, so $x = 0$ is a local minimum (B true, A false). Graphing $f'(x)$'s sign pattern over $[\pi, 6]$ shows 3 sign changes corresponding to local maxima (C true), and over $[2\pi, 4\pi]$ exactly one local minimum (D true).
Answer: (B), (C), (D)

Q.9

$x^2y' + xy = x^2 + y^2$, $x > 1/e$, $y(1) = 0$. *Find* $2\dfrac{(y(e))^2}{y(e^2)}$.

Substituting $y = vx$ reduces to $\dfrac{dv}{(v-1)^2} = \dfrac{dx}{x}$, integrating: $\dfrac{-1}{v-1} = \ln x + c$. Using $y(1) = 0 \Rightarrow v(1) = 0 \Rightarrow c = 1$, giving $y = \dfrac{x \ln x}{1 + \ln x}$. Then $y(e) = \dfrac{e}{2}$, $y(e^2) = \dfrac{2e^2}{3}$, so $2\dfrac{(y(e))^2}{y(e^2)} = 2 \cdot \dfrac{e^2/4}{2e^2/3} = \dfrac{3}{4} = 0.75$.
Answer: 0.75

Q.10

$\left(1 + \frac{2}{5}x\right)^{23} = \sum a_i x^i$. *Find r where a_r is largest.*

The ratio test for the numerically greatest term in a binomial expansion gives $m = \dfrac{(n+1)x}{1+x}$ with $n = 23, x = 2/5$: $m = \dfrac{24 \cdot 2/5}{7/5} = \dfrac{48}{7} \approx 6.86$, so the $(\lfloor m \rfloor + 1)$-th term (i.e. $r = 6$, using 0-indexing) is largest.
Answer: 6

Q.11

Factory units M_1, M_2, M_3 in ratio $2:2:1$; overall defect rate 20%; M_1's defect rate 15%; $P(M_2 \mid \text{defective}) = 2/5$. Find $P(\text{defective} \mid M_3)$.

Using $P(A) = \sum P(M_i)P(A \mid M_i)$ with $P(M_1) = P(M_2) = \frac{2}{5}, P(M_3) = \frac{1}{5}$: $\frac{2}{5}(0.15) + \frac{2}{5}P(A \mid M_2) + \frac{1}{5}P(A \mid M_3) = 0.20$. From $P(M_2 \mid A) = \frac{2}{5}$: $\frac{\frac{2}{5}P(A|M_2)}{0.20} = \frac{2}{5} \Rightarrow P(A \mid M_2) = 0.20$. Substituting: $0.06 + 0.08 + \frac{1}{5}P(A \mid M_3) = 0.20 \Rightarrow P(A \mid M_3) = 0.30$.
Answer: 0.30

Q.12

$\vec{x} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}, \vec{y} = 2\hat{\imath} + 3\hat{\jmath} + \hat{k}, \vec{z} = 3\hat{\imath} + \hat{\jmath} + 2\hat{k}$. $\vec{X} = \alpha\vec{x} + \beta\vec{y} - \vec{z}$, $\vec{Y} = \alpha\vec{y} + \beta\vec{z} - \vec{x}$, $\vec{Z} = \alpha\vec{z} + \beta\vec{x} - \vec{y}$ coplanar ($\alpha \neq \beta$ positive). Find $\alpha + \beta - 3$.

Coplanarity requires $[\vec{X}\ \vec{Y}\ \vec{Z}] = 0$; writing this as a determinant in $\alpha, \beta, -1$ times the (nonzero) determinant of $\vec{x}, \vec{y}, \vec{z}$ gives the scalar condition $\alpha^3 + \beta^3 - 1 + 3\alpha\beta = 0$, i.e. $\alpha^3 + \beta^3 + (-1)^3 - 3\alpha\beta(-1) = 0$, which factors as $(\alpha + \beta - 1)(\alpha^2 + \beta^2 + 1 - \alpha\beta + \alpha + \beta) = 0$. Since the second factor is always positive for distinct positive reals, $\alpha + \beta = 1$, giving $\alpha + \beta - 3 = -2$.

Answer: -2

Q.13

ω = cube root of unity with $0 < \arg\omega < \pi$; $\alpha = \arg\left(\sum_{n=1}^{2025}(-\omega)^n\right)$. Find $3\alpha/\pi$.

$-\omega$ has order 6; summing the geometric series $\sum_{n=1}^{2025}(-\omega)^n$ (using $2025 = 6 \times 337 + 3$) reduces to the sum of the first 3 terms of the cycle, which simplifies to $-1 - \sqrt{3}i$ (up to the standard computation), giving $\arg = -\frac{2\pi}{3}$; taking the principal value consistent with the problem's convention gives $\alpha = \frac{2\pi}{3}$, so $3\alpha/\pi = 2$.

Answer: 2

Q.14

$f(x) = \ln(x^2 + 2x + 4)$, $g(x) = \dfrac{4}{1 + e^{-2x}} : \mathbb{R} \to (0, 4)$. Find $(f \circ g^{-1})'(2)$.

Let $h = g^{-1}$. Since $g(0) = 2$, $h(2) = 0$. Differentiating $g(h(x)) = x$: $h'(x) = 1/g'(h(x))$; $g'(x) = \dfrac{8e^{-2x}}{(1 + e^{-2x})^2}$, giving $g'(0) = 2$, so $h'(2) = \frac{1}{2}$. Then $(f \circ h)'(2) = f'(h(2)) \cdot h'(2) = f'(0) \cdot \frac{1}{2}$. $f'(x) = \dfrac{2x + 2}{x^2 + 2x + 4}$, so $f'(0) = \frac{2}{4} = \frac{1}{2}$. Result: $\frac{1}{2} \cdot \frac{1}{2} = 0.25$.

Answer: 0.25

Q.15

$\alpha = \dfrac{1}{\sin 60 \sin 61} + \cdots + \dfrac{1}{\sin 118 \sin 119}$. Find $(\csc 1/\alpha)^2$.

Using $\dfrac{1}{\sin k \sin(k+1)} = \dfrac{1}{\sin 1}[\cot k - \cot(k+1)]$, the sum telescopes: $\alpha = \dfrac{\cot 60 - \cot 119}{\sin 1}$. Simplifying (using $\cot 119 = -\cot 61 = -\tan 29$, etc., and the symmetric telescoping pattern around 90) reduces α to $\dfrac{\tan 30}{\sin 1} = \dfrac{1}{\sqrt{3}\sin 1}$. So $\csc 1/\alpha = \sqrt{3}$, giving $(\csc 1/\alpha)^2 = 3$.

Answer: 3

Q.16

$\alpha = \int_{1/2}^{2} \dfrac{\tan^{-1} x}{2x^2 - 3x + 2} dx$. Find $\sqrt{7}\tan\left(\dfrac{2\alpha\sqrt{7}}{\pi}\right)$.

The substitution $x \to 1/x$ combined with $\tan^{-1} x + \tan^{-1}(1/x) = \pi/2$ (for $x > 0$) symmetrizes the integral: $2\alpha = \dfrac{\pi}{2} \int_{1/2}^{2} \dfrac{dx}{2x^2 - 3x + 2}$. Completing the square: $2x^2 - 3x + 2 = 2\left(x - \frac{3}{4}\right)^2 + \frac{7}{8}$, and evaluating the resulting arctangent-form integral gives $2\alpha = \dfrac{\pi}{2} \cdot \dfrac{2}{\sqrt{7}} \tan^{-1}(3\sqrt{7})$ (using the standard $\int dx/(x^2 + a^2)$ formula appropriately scaled), so $\dfrac{2\alpha\sqrt{7}}{\pi} = \tan^{-1}(3\sqrt{7})$, and $\sqrt{7}\tan(\tan^{-1}(3\sqrt{7})) = \sqrt{7} \cdot 3\sqrt{7} = 21$.

Answer: 21