Arithmetic Progression (A.P)
&
Geometric Progression (G.P.)

Total no: of lectures
3

Lecture-I

1, 2, 3, 4, 5, 6, 7, ...
2, 5, 8, 11, 14, 17, ...
3, 10, 17, 24, 31, ...
0, −5, −10, −15, ...
−1, −9, −17, −25, ...
5, 5, 5, 5, ...

A.P.

$a_1, a_2, a_3, \ldots, a_{n-1}, a_n$ is said to be in A.P if & only if

$$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots = a_n - a_{n-1} = d$$

$d \Rightarrow$ Common difference

Note:
* $d > 0 \rightarrow$ increasing A.P.
* $d < 0 \rightarrow$ decreasing A.P.
* $d = 0 \rightarrow$ Constant A.P.

2, 4, 8, 16, 32, ...
3, 9, 27, 81, 243, ...
$\frac{1}{4}, \frac{1}{16}, \frac{1}{64}, \frac{1}{256}, \ldots$

G.P.

$a_1, a_2, a_3, \ldots, a_{n-1}, a_n$ is said to be in G.P. if & only if

$$\frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} = \cdots = \frac{a_n}{a_{n-1}} = r.$$

where:- r is Common Ratio.

Note: * $a, a, a, \ldots$ $\rightarrow$ A.P (d=0), $\rightarrow$ G.P (r=1) $(a \neq 0)$

* $0, 0, 0, \ldots \rightarrow$ A.P.
  ↳ not G.P.

$2, \quad 5, \quad 8, \quad 11, \quad 14, \ldots$ $\xrightarrow{}$ first term.

Let $a = 2$; $d = 3$

$2, \quad 2+3, \quad 2+2(3), \quad 2+3(3), \quad 2+4(3), \ldots$

$a, \quad a+d, \quad a+2d, \quad a+3d, \quad a+4d, \quad a+5d, \ldots, \quad a+(n-1)d$

$T_1 \quad T_2 \quad T_3 \quad T_4 \quad T_5 \quad T_6 \quad \ldots \quad T_n$

$T_{10} = a + 9d \qquad T_{71} = a + 70d$

$T_{100} = a + 99d \qquad T_{29} = a + 28d$

$$\boxed{\# \quad T_n = a + (n-1)d}$$

$\rightarrow$ General term of an A.P.

where:- $a \Rightarrow$ first term

$d \Rightarrow$ common difference.

$n \Rightarrow$ no. of terms.

$$
\begin{array}{ccccccc}
2, & 4, & 8, & 16, & 32, & - - - - & \\
\downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \\
2, & 2\times 2, & 2\times 2^2, & 2\times 2^3, & 2\times 2^4, & - - - - & \\
\downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \\
a, & ar, & ar^2, & ar^3, & ar^4, & - - -, & ar^{n-1} \\
\downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \downarrow \\
T_1 & T_2 & T_3 & T_4 & T_5 & & T_n
\end{array}
$$

Let $a = 2$, $r = 2$

$T_{10} = ar^{9}$ $\quad T_{29} = ar^{28}$

$T_{100} = ar^{99}$ $\quad T_{75} = ar^{74}$

$$\boxed{\# \; T_n = ar^{n-1}}$$

→ General term of a G.P.

where: a → first term

r → Common ratio

n → no. of terms.

①

$$
\begin{aligned}
&S = 1+2+3+4+\dots+100.\\
+\ &S = 100+99+98+97+\dots+1\\
\hline
&2S = 101+101+101+101+\dots+101
\end{aligned}
$$

$$2S = (100)(101) \Rightarrow S = \frac{(100)(101)}{2} = 50(101) = 5050.$$

②

$$
\begin{aligned}
&S = 1+2+3+\dots+(n-1)+n.\\
+\ &S = n+(n-1)+(n-2)+\dots+2+1\\
\hline
&2S = (n+1)+(n+1)+(n+1)+\dots+(n+1)
\end{aligned}
$$

$$2S = n(n+1) \Rightarrow \boxed{S = \frac{n(n+1)}{2}} \rightarrow$$ Sum of first n natural no:-

$$
\begin{aligned}
&S_n = a + (a+d) + (a+2d) + \dots + (a+(n-1)d)\\
+\ &S_n = (a+(n-1)d) + (a+(n-2)d) + (a+(n-3)d) + \dots + a\\
\hline
&2S_n = [2a+(n-1)d] + [2a+(n-1)d] + [2a+(n-1)d] + \dots + [2a+(n-1)d]
\end{aligned}
$$

$$2S_n = n[2a+(n-1)d]$$

$$\Rightarrow \boxed{S_n = \frac{n[2a+(n-1)d]}{2}}$$

→ Sum of first n terms of an A.P.

where:- $a \rightarrow$ first term

$d \rightarrow$ Common difference

$n \rightarrow$ no. of terms.

$$
\begin{aligned}
S_n &= a + \cancel{ar} + \cancel{ar^2} + \cdots + \cancel{ar^{n-2}} + \cancel{ar^{n-1}} \\
-\; rS_n &= \quad\;\; \cancel{ar} + \cancel{ar^2} + ar^3 + \cdots + ar^{n-1} + ar^n
\end{aligned}
$$

$$S_n - rS_n = a - ar^n \Rightarrow S_n(1-r) = a(1-r^n)$$

$$\Rightarrow \boxed{S_n = \frac{a(1-r^n)}{(1-r)} = \frac{a(r^n-1)}{(r-1)}}$$

Sum of first n terms of a G.P.

Note: 1) If a, b, c are in A.P then $b-a = c-b$

or

$2b = a+c$

2) If a, b, c are in G.P then $\frac{b}{a} = \frac{c}{b}$

or

$b^2 = ac$

Illustration.

1) Find the value of x for which the numbers $(5x+2)$, $(4x-1)$ & $(x+2)$ are in A.P.

Sol^n:- $(4x-1)-(5x+2) = (x+2)-(4x-1)$

$\Rightarrow 4x-1-5x-2 = x+2-4x+1$

$-x-3 = -3x+3 \Rightarrow 2x = 6 \Rightarrow \boxed{x=3}$ Ans

2) If $(K-3)$, $(2K+1)$ & $(4K+3)$ are three consecutive terms of an A.P. find the value of K.

Sol^n:- $(2K+1)-(K-3) = (4K+3)-(2K+1)$

$\Rightarrow K+4 = 2K+2 \Rightarrow 4-2 = K \Rightarrow K=2$ Ans

3) The n^{th} term of an A.P is given by $T_n = 4n-7$.
find
i) first term
ii) Common difference
iii) 20th term.

$a = T_1 = 4(1)-7 = -3$

$T_2 = 4(2)-7 = 1$

$\{d = T_2 - T_1 = 1-(-3) = 4\}$

$T_{20} = 4(20)-7 = 73.$

use for subjective question.

Note:- If $T_n = pn + q$ then common difference $= p$.

for e.g.: $T_n = 5n - 3 \Rightarrow d = 5$
$T_n = 13 - 4n \Rightarrow d = -4$
$T_n = 100n - 1 \Rightarrow d = 100$.

$\Rightarrow$ use only for MCQ.

4) The 4^{th} & 10^{th} terms of an A.P are 13 & 25 respectively. Find i) First term ii) Common diff. iii) 17^{th} term.

$\hookrightarrow 7$ $\hookrightarrow 2$

Sol^n:- Let the first term be a & common diff be d.

$T_4 = 13 \rightarrow a + 3d = 13$
$T_{10} = 25 \rightarrow a + 9d = 25$

$\because \{ T_n = a + (n-1)d \}$

$$\begin{aligned} a + 9d &= 25 \\ -\ a + 3d &= 13 \\ \hline 6d &= 12 \Rightarrow d = 2 \end{aligned}$$

$a + 9(2) = 25$
$a = 7$

$T_{17} = a + 16d = 7 + 16(2) = \underline{39}$.

5) The sum of the 5^{th} & 9^{th} terms of an A.P is 26 & the sum of its 7^{th} & 11^{th} terms is 42. Find the first three terms of the A.P.

$\{\because T_n = a+(n-1)d\}$

Solⁿ:- $T_5 + T_9 = 26 \rightarrow (a+4d)+(a+8d) = 26$

$T_7 + T_{11} = 42 \rightarrow (a+6d)+(a+10d) = 42$

$$\begin{array}{r} 2a+12d = 26 \\ -\ \ 2a+16d = 42 \\ \hline -4d = -16 \end{array} \Rightarrow \boxed{d=4}$$

$2a + 12(4) = 26$

$2a = -22 \Rightarrow \boxed{a=-11}$

$T_1 = -11$
$T_2 = -7$
$T_3 = -3$ } Ans

6) How many terms are there in the given A.P. 9, 13, 17, 21, ... 97?

Solⁿ:- $a=9,\ d=4,\ T_n = 97.$

$a+(n-1)d = 97$

$\Rightarrow 9+(n-1)4 = 97 \Rightarrow 4(n-1) = 88$

$\Rightarrow (n-1) = 22 \Rightarrow \boxed{n=23}$ Ans

7) Which term of the A.P. 45, 42, 39, 36, ... is the first negative term?

Sol^n: Let T_n be the first negative term for some n.

$\Rightarrow T_n < 0 \Rightarrow a + (n-1)d < 0$

$\Rightarrow 45 + (n-1)(-3) < 0$

$\Rightarrow 45 - 3n + 3 < 0$

$\Rightarrow 48 < 3n \Rightarrow \boxed{16 < n}$

$\Rightarrow n = 17 \Rightarrow$ first negative term $= T_{17}$

$T_{17} = a + 16d = 45 + 16(-3) = -3$

* $T_{16} = a + 15d = 45 + 15(-3) = 0.$

Lecture-3

Note: Consider three consecutive terms of an A.P. as $a-d, \ a, \ a+d.$

8) Find three numbers in A.P. whose sum is 15 & the product is 80.

Sol^n: Let the numbers be $a-d$, a & $a+d$.

Given: $a-d+a+a+d = 15 \Rightarrow 3a = 15 \Rightarrow \boxed{a=5}$

& $(a-d)(a)(a+d) = 80 \Rightarrow (5-d)(5)(5+d) = 80$

$\Rightarrow (25-d^2) = 16 \Rightarrow d^2 = 9 \Rightarrow \boxed{d = \pm 3}$

* $a=5$ & $d=3 \rightarrow 2, 5, 8$
* $a=5$ & $d=-3 \rightarrow 8, 5, 2$

Required numbers.

9) The 4^{th} term of an A.P. is 22 & 15^{th} term is 66. Find

(a) first term
(b) Common difference.
(c) S_8 (Sum of first 8 terms of an A.P.)

Sol:

$T_4 = 22 \rightarrow a+3d = 22$

$T_{15} = 66 \rightarrow a+14d = 66$

$-11d = -44 \rightarrow d = 4$

$a + 12 = 22$

$a = 10$.

$\because S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore S_8 = \frac{8}{2}[2\times10 + (8-1)4] = 4[20+28]$

$= 192$ Ans

Recall: For G.P.

$T_n = ar^{n-1}$, $S_n = \frac{a(r^n - 1)}{r-1} = \frac{a(1-r^n)}{(1-r)}$

$a, b, c \rightarrow$ G.P

$\frac{b}{a} = \frac{c}{b} \Rightarrow b^2 = ac$

10) For what values of x, the numbers $(x+9)$, $(x-6)$ & 4 are in G.P.?

Sol^n: $x+9,\ x-6$ & $4 \rightarrow$ G.P $\Rightarrow \dfrac{x-6}{x+9} = \dfrac{4}{x-6}$

$\Rightarrow (x-6)^2 = 4(x+9) \Rightarrow x^2 - 12x + \cancel{36} = 4x + \cancel{36} \Rightarrow x^2 - 16x = 0$

$\Rightarrow x(x-16) = 0 \Rightarrow x = 0$ or $x = 16$ Ans

11) Which term of the G.P 5, 10, 20, 40, ... is 2560?

Sol^n: $a = 5,\ r = 2,\ T_n = 2560 \Rightarrow ar^{n-1} = 2560$

$\Rightarrow 5(2)^{n-1} = 2560 \Rightarrow 2^{n-1} = \dfrac{2560}{5} = 512$

$\Rightarrow 2^{n-1} = 2^9 \Rightarrow n-1 = 9 \Rightarrow n = 10$

* $2^1 = 2$	$2^6 = 64$	$3^1 = 3$	$3^6 = 729$	$5^1 = 5$	$7^1 = 7$
$2^2 = 4$	$2^7 = 128$	$3^2 = 9$		$5^2 = 25$	$7^2 = 49$
$2^3 = 8$	$2^8 = 256$	$3^3 = 27$		$5^3 = 125$	$7^3 = 343$
$2^4 = 16$	$2^9 = 512$	$3^4 = 81$		$5^4 = 625$	
$2^5 = 32$	$2^{10} = 1024$	$3^5 = 243$		$5^5 = 3125$	

12) If the 4^{th} & 7^{th} terms of a G.P are 54 & 1458 respectively, then find its i) r ii) a iii) T_n iv) T_6.

Sol^n: $T_4 = 54 \longrightarrow ar^3 = 54 \rightarrow ①$

$T_7 = 1458 \longrightarrow ar^6 = 1458 \rightarrow ②$

$\dfrac{②}{①} \Rightarrow \dfrac{\cancel{a}r^6}{\cancel{a}r^3} = \dfrac{1458}{54}$

$\Rightarrow r^3 = 27 \Rightarrow \boxed{r = 3} \rightarrow a(3)^3 = 54 \Rightarrow \boxed{a = 2}$

* $T_n = ar^{n-1} = 2(3)^{n-1}$ * $T_6 = 2(3)^5 = 486.$

Note: Consider three terms of a G.P to be

$$\frac{a}{r},\ a,\ ar.$$

13) The sum of three numbers in G.P. is $\frac{39}{10}$ & their product is 1. Find the numbers.

Sol^n: Let the no: be $\frac{a}{r}, a$ & ar.

$$\frac{a}{\cancel{r}} \cdot a \cdot a\cancel{r} = 1 \Rightarrow a^3 = 1 \Rightarrow \boxed{a=1}$$

$$\frac{a}{r} + a + ar = \frac{39}{10} \Rightarrow \frac{1}{r} + 1 + r = \frac{39}{10} \Rightarrow \frac{1+r+r^2}{r} = \frac{39}{10}$$

$$\Rightarrow 10 + 10r + 10r^2 = 39r \Rightarrow 10r^2 - 29r + 10 = 0$$

$$\Rightarrow 10r^2 - 25r - 4r + 10 = 0 \Rightarrow 5r(2r-5) - 2(2r-5) = 0$$

$$\Rightarrow (2r-5)(5r-2) \Rightarrow r = 5/2 \text{ or } r = \frac{2}{5}.$$

* $a=1,\ r=5/2 \rightarrow \frac{2}{5}, 1, 5/2$ **Ans**

'or'

** $a=1,\ r=\frac{2}{5} \rightarrow 5/2, 1, \frac{2}{5}$ **Ans**

14) Find the sum of 8 terms of the G.P. $\frac{1}{3}, \frac{1}{6}, \frac{1}{12}, \frac{1}{24}, \dots$

$a = \frac{1}{3}, \quad r = \frac{1}{2}$

$\because \; S_n = \frac{a(1-r^n)}{1-r} \qquad \therefore \; S_8 = \frac{\frac{1}{3}\left[1-\left(\frac{1}{2}\right)^8\right]}{1-\frac{1}{2}}$

$\Rightarrow S_8 = \frac{2}{3}\left[1 - \frac{1}{256}\right] = \frac{\cancel{2}}{\cancel{3}}\left[\frac{\overset{85}{\cancel{255}}}{\underset{128}{\cancel{256}}}\right] = \frac{85}{128}$ Ans