Matrices

$\boxed{\text{Lecture-I}}$

Total no. of lectures = 2

↳ Rectangular arrangement of numbers in rows & columns.

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2}$, $A = \begin{bmatrix} 1 \\ 2 \end{bmatrix}_{2\times 1}$, $A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}_{1\times 3}$

↓ square Matrix

↳ Column matrix $(m\times 1)$

↳ Row matrix $(1\times n)$

$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times 3}$, $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}_{3\times 2}$

$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ ↳ Null or zero Matrix

$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ↳ Identity or unit matrix.

Identity Matrix (only exist for square matrix)

↳ B is said to be Identity matrix if

$AB = BA = A$.

& we denote B as "I"

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$A_{m \times p} \, B_{p \times n} = C_{m \times n}$$

* $A_{2\times3} \, B_{3\times1} = C_{2\times1}$ ✓

* $A_{3\times4} \, B_{4\times5} = C_{3\times5}$ ✓

* $A_{1\times5} \, B_{5\times1} = C_{1\times1}$ ✓

* $A_{4\times5} \, B_{4\times5} = \times$

$$\begin{bmatrix} 5 & -3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & -3 \\ -1 & 2 \end{bmatrix}$$

Identity matrix.

9A) 4) Construct a (3×2) matrix $[a_{ij}]_{3\times2}$

for which $a_{ij} = (i \times j)$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}_{3\times2} = \begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \end{bmatrix}$$

Important Questions.

1) Find x & y, if $\begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 2x \\ 1 \end{bmatrix} + 2\begin{bmatrix} -4 \\ 5 \end{bmatrix} = 4\begin{bmatrix} 2 \\ y \end{bmatrix}$ [2003]

Solⁿ: $\begin{bmatrix} 6x-2 \\ -2x+4 \end{bmatrix} + \begin{bmatrix} -8 \\ 10 \end{bmatrix} = \begin{bmatrix} 8 \\ 4y \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 6x-10 \\ -2x+14 \end{bmatrix} = \begin{bmatrix} 8 \\ 4y \end{bmatrix} \Rightarrow$ $6x-10 = 8 \Rightarrow 6x = 18 \Rightarrow \boxed{x=3}$ Ans

$-2x+14 = 4y \Rightarrow -6+14 = 4y$

$\Rightarrow 8 = 4y \Rightarrow \boxed{y=2}$ Ans

2) Given $\begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix} \cdot X = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$ Find (i) order of the Matrix X
(ii) the Matrix X.

$A_{2\times 2} \; X_{2\times 1} = B_{2\times 1}$ [2012]

(i) order of the Matrix $X = 2\times 1$.

(ii) Let $X = \begin{bmatrix} a \\ b \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2a+b \\ -3a+4b \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix} \Rightarrow$ $2a+b = 7 \xrightarrow{\times 4} 8a+4b = 28$

$-3a+4b = 6$ $\qquad -3a+4b = 6$

$11a = 22 \Rightarrow a = 2$

$2\times 2 + b = 7 \Rightarrow b = 3$

$$\Rightarrow X = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$ Ans

12) Given $A = \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} -4 \\ 5 \end{bmatrix}$ & $D = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

find $AB + 2C - 4D$.

Solⁿ:

$$AB + 2C - 4D = \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 6 \\ 1 \end{bmatrix} + 2\begin{bmatrix} -4 \\ 5 \end{bmatrix} - 4\begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 16 \\ -2 \end{bmatrix} + \begin{bmatrix} -8 \\ 10 \end{bmatrix} - \begin{bmatrix} 8 \\ 8 \end{bmatrix}$$

$$= \begin{bmatrix} 8 \\ 8 \end{bmatrix} - \begin{bmatrix} 8 \\ 8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$ Ans

13) Given $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ & $B = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$. find

$A^2 + AB + B^2$

Solⁿ:

$$A^2 = A \cdot A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix}$$

$$B^2 = B \cdot B = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 6 \\ -2 & -3 \end{bmatrix}$$

$$A^2 + AB + B^2 = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ -2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 9 \\ 5 & 4 \end{bmatrix}$$ Ans

14) Q:- i) find $A^2 - A + BC$

ii) find $AC + B^2 - 10C$.

<u>Transpose of a Matrix</u> $\underbrace{(A^T \text{ or } A' \text{ or } A^t)}$ → Notation.

If $A = [a_{ij}]_{m \times n}$ then $A^T = [a_{ji}]_{n \times m}$

<u>Ex:</u> $A = \begin{bmatrix} 1 & -5 \\ 3 & 4 \end{bmatrix} \longrightarrow A^T = \begin{bmatrix} 1 & 3 \\ -5 & 4 \end{bmatrix}$

(9c)

15) If $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$ & I is the identity matrix of the same order & A^T is the transpose of A.

Find $A^TB + BI$

Sol^n: Here, $A^T = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$, $A^TB = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$

$A^TB = \begin{bmatrix} 7 & -1 \\ 17 & -1 \end{bmatrix}$, $BI = B = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$

{$\because$ I is an identity matrix}

$A^TB + BI = \begin{bmatrix} 7 & -1 \\ 17 & -1 \end{bmatrix} + \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 11 & -3 \\ 16 & 2 \end{bmatrix}$

Exercises - 9C

35) $A = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$ & $AB = A+B$.

$a = ?$
$b = ?$
$c = ?$

$$AB = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} a & b \\ 0 & c \end{bmatrix} = \begin{bmatrix} 3a & 3b \\ 0 & 2c \end{bmatrix}$$

$$A+B = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} = \begin{bmatrix} 3+a & b \\ 0 & c+2 \end{bmatrix}$$

$\Rightarrow 3a = 3+a$

$a = 3/2$

$3b = b$

$2b = 0 \Rightarrow b = 0$

$2c = c+2$

$c = 2$

Imp.

37) $A = \begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix}$ & $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ & $A^2 = 9A + mI$.

$m = ?$

$$A^2 = \begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ -9 & 49 \end{bmatrix}$$

$$9A = 9\begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix} = \begin{bmatrix} 18 & 0 \\ -9 & 63 \end{bmatrix}$$

$\because A^2 = 9A + mI \Rightarrow mI = A^2 - 9A$

$$\Rightarrow m\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ -9 & 49 \end{bmatrix} - \begin{bmatrix} 18 & 0 \\ -9 & 63 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} = \begin{bmatrix} -14 & 0 \\ 0 & -14 \end{bmatrix} \Rightarrow m = -14$$