Lecture-I

Proportion.

Total Lecture-3.

Four non-zero quantities a, b, c & d are said to be in proportion if $a:b = c:d$ $\left\{\frac{a}{b} = \frac{c}{d}\right\}$ and we write

$$\underbrace{a:b :: c:d}$$

↳ Read as a is to b as c is to d.

* $a:b :: c:d$

first term, second term, third term, fourth term

** d is called the fourth proportional to a, b, c.

*** a & d are called extremes.

b & c are called means.

Note:- $a:b :: c:d \Rightarrow \frac{a}{b} = \frac{c}{d} \Rightarrow ad = bc$

$\Rightarrow$ Product of extremes = product of means.

Continued proportion.

We say a, b, c, d, e, f, g etc are in Continued proportion, if $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \frac{d}{e} = \frac{e}{f} = \frac{f}{g}$.

Mean Proportion (Or Geometric Mean)

Let a, b & c be in Continued proportion

$\frac{a}{b} = \frac{b}{c}$ 'or' $b^2 = ac$ 'or' $b = \sqrt{ac}$

Here, b is called Mean proportion or Geometric mean of a & c.

Third Proportional.

If $a:b = b:c$ then c is called the third proportional to a & b.

* c is fourth proportional a, b, b.

Illustration.

1) What least number must be added to each of the numbers 6, 15, 20 & 43, so that the resulting numbers are proportional.

Sol^n: Let the required number to be added is x.

$6+x,\ 15+x,\ 20+x$ & $43+x \longrightarrow$ proportion

$\Rightarrow \dfrac{6+x}{15+x} = \dfrac{20+x}{43+x} \Rightarrow (6+x)(43+x) = (20+x)(15+x)$

$\Rightarrow \cancel{x^2} + 49x + 258 = \cancel{x^2} + 35x + 300$

$\Rightarrow 14x = 42 \Rightarrow \boxed{x=3}$

2) Find:

i) The fourth proportional to 7, 13 & 35

ii) third proportional to 9 & 15

iii) mean proportion between 5 & 80.

Sol^n: i) $7:13::35:x \longrightarrow \dfrac{7}{13} = \dfrac{35}{x} \Rightarrow x = \dfrac{35\times 13}{7} \Rightarrow \boxed{x=65}$

ii) $9:15::15:x \Rightarrow \dfrac{9}{15} = \dfrac{15}{x} \Rightarrow x = \dfrac{15\times 15}{9} \Rightarrow \boxed{x=25}$

iii) $5:x::x:80 \Rightarrow \dfrac{5}{x} = \dfrac{x}{80} \Rightarrow x^2 = 400 \Rightarrow \boxed{x=20}$

3) If b is the mean proportion of a & c, Prove that

$$\frac{a^2-b^2+c^2}{a^{-2}-b^{-2}+c^{-2}} = b^4.$$

Sol^n: $\because$ b is the mean proportion of a & c

$\therefore$ $b^2 = ac.$

$$L.H.S = \frac{a^2-b^2+c^2}{\frac{1}{a^2}-\frac{1}{b^2}+\frac{1}{c^2}} = \frac{(a^2-b^2+c^2)}{\frac{b^2c^2-a^2c^2+a^2b^2}{a^2b^2c^2}}$$

$$= \frac{(a^2-b^2+c^2)(a^2b^2c^2)}{b^2c^2-a^2c^2+a^2b^2} = \frac{(a^2-ac+c^2)(a^2(ac)c^2)}{(ac)c^2-a^2c^2+a^2\cdot ac}$$

$$= \frac{\cancel{(a^2-ac+c^2)}(a^3c^3)}{(ac)\cancel{(c^2-ac+a^2)}} = a^2c^2 = (ac)^2 = (b^2)^2 = b^4 = R.H.S.$$

4) If x, y, z are in Continued Proportion, prove that

$$\frac{(x+y)^2}{(y+z)^2} = \frac{x}{z}.$$

Sol^n: $\because$ x, y & z are in Continued proportion.

$\therefore$ $\frac{x}{y} = \frac{y}{z} \Rightarrow \boxed{y^2 = xz}$ or $\boxed{y = \sqrt{xz}}$

$$L.H.S = \frac{x^2+y^2+2xy}{y^2+z^2+2yz} = \frac{x^2+xz+2x(\sqrt{xz})}{xz+z^2+2(\sqrt{xz})z} = \frac{x\cancel{(x+z+2\sqrt{xz})}}{z\cancel{(x+z+2\sqrt{xz})}}$$

$$= \frac{x}{z} = R.H.S.$$

$\boxed{\text{Lecture-II}}$

K-Method.

* $a:b::c:d \Rightarrow \frac{a}{b}=\frac{c}{d}=K \Rightarrow \boxed{c=Kd}$ & $\boxed{a=Kb}$

** a,b,c & d are in Continued proportion

$$\frac{a}{b}=\frac{b}{c}=\frac{c}{d}=K$$

$\Rightarrow \boxed{c=Kd}$

$\Rightarrow \boxed{b=K^2d}$

$\Rightarrow \boxed{a=K^3d}$

$b=Kc$

$b=K(Kd)$

$a=Kb$

$a=K(K^2d)$

*** $\frac{x}{a}=\frac{y}{b}=\frac{z}{c}=\frac{w}{d}=K \Rightarrow \left.\begin{aligned} x&=Ka\\ y&=Kb\\ z&=Kc\\ w&=Kd\end{aligned}\right\}$

**** a,b,c are in Continued prop.

$$\frac{a}{b}=\frac{b}{c}=K \Rightarrow \boxed{\begin{aligned} b&=Kc\\ a&=K^2c\end{aligned}}$$

Note: $K^4+K^2+1=(K^2-K+1)(K^2+K+1)$

Pf: $R.H.S=[(K^2+1)-K][(K^2+1)+K]=(K^2+1)^2-K^2$

$=K^4+2K^2+1-K^2=K^4+K^2+1=L.H.S.$

5) If a, b, c are in continued proportion, prove that

(a) $$\frac{a+b+c}{a-b+c} = \frac{(a+b+c)^2}{(a^2+b^2+c^2)}$$

Pf: $a, b, c \rightarrow$ continued proportion $\Rightarrow \frac{a}{b} = \frac{b}{c} = K$

$\Rightarrow b = KC$ & $a = K^2C$

$$L.H.S = \frac{K^2C + KC + C}{K^2C - KC + C} = \frac{C(K^2+K+1)}{C(K^2-K+1)} = \frac{(K^2+K+1)}{(K^2-K+1)}$$

$$R.H.S = \frac{(K^2C + KC + C)^2}{(K^4C^2 + K^2C^2 + C^2)} = \frac{C^2(K^2+K+1)^2}{C^2(K^4+K^2+1)} = \frac{\cancel{C^2}(K^2+K+1)^{\cancel{2}}}{\cancel{C^2}\cancel{(K^2+K+1)}(K^2-K+1)}$$

$$= \frac{K^2+K+1}{K^2-K+1}$$

$L.H.S = R.H.S.$

$$(K^2C + KC + C)^2 = [(C)(K^2+K+1)]^2$$

$$= (C)^2(K^2+K+1)^2$$

Results.

1) $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{b}{a} = \frac{d}{c}$ (Invertendo)

2) $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a}{c} = \frac{b}{d}$ (Alternendo)

3) $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{b} = \frac{c+d}{d}$ (Componendo)

4) $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a-b}{b} = \frac{c-d}{d}$ (Dividendo)

5) $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}$ (Componendo & Dividendo).

$\frac{a}{b} = \frac{c}{d} \rightarrow \frac{a}{b}+1 = \frac{c}{d}+1 \Rightarrow \frac{a+b}{b} = \frac{c+d}{d} \rightarrow *$

$\rightarrow \frac{a}{b}-1 = \frac{c}{d}-1 \Rightarrow \frac{a-b}{b} = \frac{c-d}{d} \rightarrow **$

$\frac{*}{**} \Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}$

6) If $\frac{a}{b} = \frac{c}{d}$ then prove that $\frac{3a-5b}{3a+5b} = \frac{3c-5d}{3c+5d}$.

Solⁿ:

$\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{3a}{5b} = \frac{3c}{5d}$ → Apply Componendo & Dividendo

$\Rightarrow \frac{3a+5b}{3a-5b} = \frac{3c+5d}{3c-5d}$ → Apply invertendo

$\Rightarrow \frac{3a-5b}{3a+5b} = \frac{3c-5d}{3c+5d}$.

7) If $\frac{8a-5b}{8c-5d} = \frac{8a+5b}{8c+5d}$ then Prove that $\frac{a}{b} = \frac{c}{d}$.

Solⁿ:

$\frac{8c+5d}{8c-5d} = \frac{8a+5b}{8a-5b}$ {by Alternendo} By Comp. & Divi.

$\Rightarrow \frac{(8c+5d)+(8c-5d)}{(8c+5d)-(8c-5d)} = \frac{(8a+5b)+(8a-5b)}{(8a+5b)-(8a-5b)}$

$\Rightarrow \frac{16c}{10d} = \frac{16a}{10b} \Rightarrow \frac{c}{d} = \frac{a}{b}$ H.P.

Lecture-III

$$(a+b)^3 = a^3+b^3+3ab(a+b) = a^3+b^3+3a^2b+3ab^2.$$
$$(a-b)^3 = a^3-b^3-3ab(a-b) = a^3-b^3-3a^2b+3ab^2.$$

8) Given that $\dfrac{a^3+3ab^2}{b^3+3a^2b} = \dfrac{63}{62}$. find $a:b$ using Componendo & dividendo.

Sol^n: Apply Comp. & dividendo $\rightarrow$ $\dfrac{a^3+3ab^2+b^3+3a^2b}{a^3+3ab^2-b^3-3a^2b} = \dfrac{63+62}{63-62}$

$$\Rightarrow \frac{(a+b)^3}{(a-b)^3} = \frac{125}{1} = \left(\frac{5}{1}\right)^3 \Rightarrow \boxed{\frac{a+b}{a-b} = \frac{5}{1}}$$

Apply. Comp. & dividendo.

$$\Rightarrow \frac{(a+b)+(a-b)}{(a+b)-(a-b)} = \frac{5+1}{5-1} \Rightarrow \frac{2a}{2b} = \frac{6}{4} \Rightarrow \frac{a}{b} = \frac{3}{2}$$

$$\Rightarrow \boxed{a:b = 3:2}$$ Ans

9) Using Componendo & dividendo, find the value of x when

$$\frac{\sqrt{3x+4}+\sqrt{3x-5}}{\sqrt{3x+4}-\sqrt{3x-5}} = \frac{9}{1}.$$

Sol^n: Apply Comp. & divi.

$$\frac{(\sqrt{3x+4}+\sqrt{3x-5})+(\sqrt{3x+4}-\sqrt{3x-5})}{(\sqrt{3x+4}+\sqrt{3x-5})-(\sqrt{3x+4}-\sqrt{3x-5})} = \frac{9+1}{9-1}$$

$$\Rightarrow \frac{2\sqrt{3x+4}}{2\sqrt{3x-5}} = \frac{10}{8} \Rightarrow \frac{\sqrt{3x+4}}{\sqrt{3x-5}} = \frac{5}{4} \Rightarrow \frac{3x+4}{3x-5} = \frac{25}{16}$$

$$\Rightarrow 48x+64 = 75x-125$$

$$\Rightarrow 64+125 = 75x-48x$$

$$\Rightarrow 27x = 189 \Rightarrow \boxed{x=7}$$ Ans

* $x^4+2x^2+1=(x^2+1)^2$
** $x^4-2x^2+1=(x^2-1)^2$

10) Using the properties of proportion, find the value of x when

$$\frac{x^4+1}{2x^2}=\frac{17}{8}.$$

Sol^n:- Apply Comp. & dividendo $\rightarrow$ $\dfrac{x^4+1+2x^2}{x^4+1-2x^2}=\dfrac{17+8}{17-8}$

$$\Rightarrow \frac{(x^2+1)^2}{(x^2-1)^2}=\frac{25}{9} \Rightarrow \left(\frac{x^2+1}{x^2-1}\right)^2=\left(\frac{5}{3}\right)^2 \Rightarrow \frac{x^2+1}{x^2-1}=\frac{5}{3}$$

$$\Rightarrow 3x^2+3=5x^2-5 \Rightarrow 2x^2=8 \Rightarrow x^2=4 \Rightarrow \boxed{x=\pm 2}$$

Ans

11) If $\dfrac{x}{1}=\dfrac{\sqrt{a+1}+\sqrt{a-1}}{\sqrt{a+1}-\sqrt{a-1}}$, using properties of proportion, show that $x^2-2ax+1=0$.

Sol^n:- Apply Comp. & dividendo

$$\frac{x+1}{x-1}=\frac{2\sqrt{a+1}}{2\sqrt{a-1}} \Rightarrow \frac{x+1}{x-1}=\frac{\sqrt{a+1}}{\sqrt{a-1}}$$

$$\Rightarrow \frac{(x+1)^2}{(x-1)^2}=\frac{a+1}{a-1} \Rightarrow \frac{x^2+2x+1}{x^2-2x+1}=\frac{a+1}{a-1}$$

$$\Rightarrow \frac{\cancel{2}(x^2+1)}{\cancel{4}2x}=\frac{\cancel{2}a}{\cancel{2}} \Rightarrow \frac{x^2+1}{2x}=a \Rightarrow x^2+1=2ax$$

$$\Rightarrow \boxed{x^2-2ax+1=0}$$

H.P.

H.W

7C → 2, 3, 4, 7, 8, 10, 13, 16, 17, 22

7B → 2 (iv), 3 (v), 4 (v), 6, 8, 10,
15 (ii), 16 (ii), 17 (v), 21.

Exercise - 7C.

7) If $x = \frac{6ab}{a+b}$ then P.T $\frac{x+3a}{x-3a} + \frac{x+3b}{x-3b} = 2$.

Pf:

$\frac{x}{3a} = \frac{2b}{a+b}$ & $\frac{x}{3b} = \frac{2a}{a+b}$

$\frac{x+3a}{x-3a} = \frac{3b+a}{b-a}$ — ① & $\frac{x+3b}{x-3b} = \frac{3a+b}{a-b}$ — ②

① + ② $\frac{x+3a}{x-3a} + \frac{x+3b}{x-3b} = \frac{3b+a}{b-a} - \frac{3a+b}{b-a} = \frac{3b+a-3a-b}{b-a} = \frac{2b-2a}{b-a}$

$= 2$

10) $\frac{\sqrt{x+5} + \sqrt{x-16}}{\sqrt{x+5} - \sqrt{x-16}} = \frac{7}{3}$ P.T → $x = 20$

$\Rightarrow \frac{\cancel{2}\sqrt{x+5}}{\cancel{2}\sqrt{x-16}} = \frac{10}{4}$ $\Rightarrow \frac{x+5}{x-16} = \frac{100}{16}$ $\Rightarrow 16x + 80 = 100x - 1600$

$\Rightarrow 1680 = 84x$

$\Rightarrow x = 20.$

TB

16) ii) $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$

P.T. $\frac{a^3+c^3+e^3}{b^3+d^3+f^3} = \frac{ace}{bdf}$

Sol^n:

$a=kb, \quad c=kd, \quad e=kf$

$L.H.S = \frac{k^3b^3+k^3d^3+k^3f^3}{b^3+d^3+f^3}$

$= k^3 = k \times k \times k$

$= \frac{a}{b} \times \frac{c}{d} \times \frac{e}{f} = R.H.S.$

18 (ii) $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = k \rightarrow$

$c = kd$

$b = k^2d$

$a = k^3d$

P.T $(a+d)(b+c) - (a+c)(b+d) = (b-c)^2$