Total lectures = 4.

Trigonometry.

$\boxed{\text{Lecture-I}}$

1) $\sin\theta = \dfrac{p}{h}$

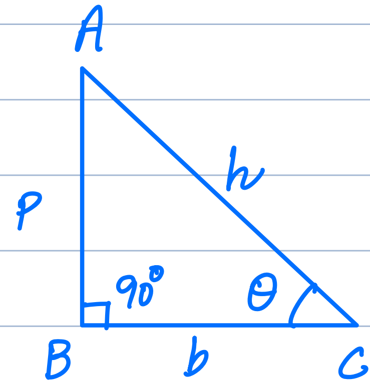
2) $\cos\theta = \dfrac{b}{h}$

3) $\tan\theta = \dfrac{p}{b}$

4) $\csc\theta = \dfrac{h}{p}$

5) $\sec\theta = \dfrac{h}{b}$

6) $\cot\theta = \dfrac{b}{p}$.

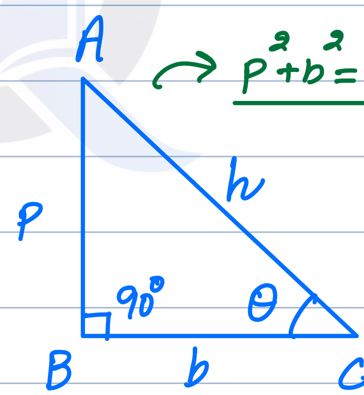


Sin → Sine.
Cos → Cosine.
tan → tangent.
Cosec → Cosecant.
Sec → Secant.
Cot → Cotangent.

Note:
1) $\dfrac{1}{\sin\theta} = \csc\theta$.
2) $\dfrac{1}{\cos\theta} = \sec\theta$.
3) $\dfrac{1}{\tan\theta} = \cot\theta$.
4) $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$.
5) $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$.

Trigonometric Identities.

1)	$\sin^2\theta + \cos^2\theta = 1$
2)	$1 + \tan^2\theta = \sec^2\theta$.
3)	$1 + \cot^2\theta = \csc^2\theta$.



$p^2 + b^2 = h^2$.

Pf: 1) $L.H.S = \dfrac{p^2}{h^2} + \dfrac{b^2}{h^2} = \dfrac{p^2+b^2}{h^2} = \dfrac{h^2}{h^2} = 1 = R.H.S$

2) $L.H.S = 1 + \dfrac{p^2}{b^2} = \dfrac{b^2+p^2}{b^2} = \dfrac{h^2}{b^2} = \left(\dfrac{h}{b}\right)^2 = (\sec\theta)^2 = R.H.S.$

Important Notes.

1) $1-\sin^2\theta = \cos^2\theta$; $1-\cos^2\theta = \sin^2\theta$.

2) $\sec^2\theta - 1 = \tan^2\theta$; $\sec^2\theta - \tan^2\theta = 1$.

3) $\operatorname{cosec}^2\theta - 1 = \cot^2\theta$; $\operatorname{cosec}^2\theta - \cot^2\theta = 1$.

4) $\sec\theta - \tan\theta = \dfrac{1}{\sec\theta + \tan\theta}$, $\sec\theta + \tan\theta = \dfrac{1}{\sec\theta - \tan\theta}$.

5) $\operatorname{cosec}\theta - \cot\theta = \dfrac{1}{\operatorname{cosec}\theta + \cot\theta}$, $\operatorname{cosec}\theta + \cot\theta = \dfrac{1}{\operatorname{cosec}\theta - \cot\theta}$.

Pf: 4) $\sec^2\theta - \tan^2\theta = 1 \longrightarrow (\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$.

* $\sec\theta - \tan\theta = \dfrac{1}{\sec\theta + \tan\theta}$.

* $\sec\theta + \tan\theta = \dfrac{1}{\sec\theta - \tan\theta}$.

Illustration.

Note: $(\sin A)(\sin A) = (\sin A)^2$
$= \sin^2 A.$

$(\sin A)(\sin A) \neq \sin^2 A^2.$

Prove the given below identities.

1) $\dfrac{\sin A}{1+\cos A} + \dfrac{1+\cos A}{\sin A} = 2\,\text{Cosec}A.$

Pf: $L.H.S = \dfrac{\sin^2 A + (1+\cos A)^2}{(1+\cos A)(\sin A)} = \dfrac{\sin^2 A + (1)^2 + \cos^2 A + 2\cos A}{(1+\cos A)(\sin A)}$

$\{\because \sin^2 A + \cos^2 A = 1\}.$

$= \dfrac{1+1+2\cos A}{(1+\cos A)(\sin A)} = \dfrac{2+2\cos A}{(1+\cos A)(\sin A)} = \dfrac{2\cancel{(1+\cos A)}}{\cancel{(1+\cos A)}(\sin A)}$

$= \dfrac{2}{\sin A} = 2\,\text{Cosec}A. \quad \left\{\because \dfrac{1}{\sin A} = \text{Cosec}A\right\}.$

2) $(\text{Cosec}A - \sin A)(\sec A - \cos A)(\sec^2 A) = \tan A.$

Pf: $L.H.S = \left(\dfrac{1}{\sin A} - \sin A\right)\left(\dfrac{1}{\cos A} - \cos A\right) \cdot \dfrac{1}{\cos^2 A} \quad \left\{\begin{array}{l}\because \text{Cosec}A = 1/\sin A \\ \sec A = 1/\cos A\end{array}\right\}$

$= \left(\dfrac{1-\sin^2 A}{\sin A}\right)\left(\dfrac{1-\cos^2 A}{\cos A}\right) \cdot \dfrac{1}{\cos^2 A}.$

$= \dfrac{\cancel{(\cos^2 A)}\,\cancel{(\sin^2 A)}}{\cancel{(\sin A)}(\cos A)\cancel{(\cos^2 A)}} \quad \left\{\begin{array}{l}\because 1-\sin^2 A = \cos^2 A \\ \& \ 1-\cos^2 A = \sin^2 A\end{array}\right\}.$

$= \dfrac{\sin A}{\cos A} = \tan A$

3) $\dfrac{1}{\operatorname{cosec}\theta - \cot\theta} - \dfrac{\cot\theta}{\cos\theta} = \cot\theta$

4)

Pf: 3) $\because \dfrac{1}{\operatorname{Cosec}\theta - \operatorname{Cot}\theta} = \operatorname{Cosec}\theta + \operatorname{Cot}\theta$

$\therefore$ L.H.S $= (\operatorname{Cosec}\theta + \operatorname{Cot}\theta) - \dfrac{\cancel{\operatorname{Cos}\theta}}{(\operatorname{Sin}\theta)(\cancel{\operatorname{Cos}\theta})}$

$(\operatorname{Cosec}\theta + \operatorname{Cot}\theta) - \operatorname{Cosec}\theta = \operatorname{Cot}\theta =$ R.H.S.

Prove that:

$$\frac{\operatorname{cosec}^2 x - \sin^2 x \cot^2 x - \cot^2 x}{\sin^2 x} = 1$$

L.H.S $= \dfrac{1 - (\cancel{\operatorname{Sin}^2 x})\dfrac{(\operatorname{Cos}^2 x)}{(\cancel{\operatorname{Sin}^2 x})}}{\operatorname{Sin}^2 x}$ $\left\{ \because \operatorname{Cosec}^2 x - \operatorname{Cot}^2 x = 1 \right\}$

$= \dfrac{\operatorname{Sin}^2 x}{\operatorname{Sin}^2 x} = 1 =$ R.H.S

3) $\dfrac{\cos A}{1-\tan A} + \dfrac{\sin A}{1-\cot A} = \cos A + \sin A.$

Pf: $L.H.S = \dfrac{\cos A}{1-\dfrac{\sin A}{\cos A}} + \dfrac{\sin A}{1-\dfrac{\cos A}{\sin A}}$ $\left\{ \because \tan A = \dfrac{\sin A}{\cos A} \text{ \& } \cot A = \dfrac{\cos A}{\sin A} \right\}$

$= \dfrac{\cos A}{\dfrac{\cos A-\sin A}{\cos A}} + \dfrac{\sin A}{\dfrac{\sin A-\cos A}{\sin A}} = \dfrac{\cos^2 A}{\cos A-\sin A} + \dfrac{\sin^2 A}{\sin A-\cos A}.$

$= \dfrac{\cos^2 A}{\cos A-\sin A} - \dfrac{\sin^2 A}{\cos A-\sin A} = \dfrac{\cos^2 A-\sin^2 A}{\cos A-\sin A}$

$= \dfrac{\cancel{(\cos A-\sin A)}(\cos A+\sin A)}{\cancel{(\cos A-\sin A)}} = \cos A+\sin A = R.H.S.$

H.P.

4) $\dfrac{\sin A}{1+\cos A} = \operatorname{cosec} A - \cot A.$

Pf: (M-I) $L.H.S = \dfrac{\sin A\,(1-\cos A)}{(1+\cos A)(1-\cos A)} = \dfrac{\sin A\,(1-\cos A)}{1-\cos^2 A}$

$\{\because 1-\cos^2 A = \sin^2 A\}$

$= \dfrac{\cancel{\sin A}\,(1-\cos A)}{\sin^{\cancel{2}} A} = \dfrac{1-\cos A}{\sin A} = \dfrac{1}{\sin A} - \dfrac{\cos A}{\sin A}$

$= \operatorname{cosec} A - \cot A$ $\left\{ \because \dfrac{1}{\sin A} = \operatorname{cosec} A \text{ \& } \dfrac{\cos A}{\sin A} = \cot A \right\}$

$= R.H.S$

H.P.

(M-II) R.H.S $= \operatorname{Cosec}\theta - \cot\theta = \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta} = \frac{(1-\cos\theta)}{\sin\theta}\frac{(1+\cos\theta)}{(1+\cos\theta)}$

$$= \frac{1-\cos^2\theta}{\sin\theta(1+\cos\theta)} = \frac{\sin^{\cancel{2}}\theta}{\cancel{\sin\theta}(1+\cos\theta)} \quad \left\{\because 1-\cos^2\theta = \sin^2\theta\right\}$$

$$= \frac{\sin\theta}{1+\cos\theta} = \text{L.H.S.}$$

$\boxed{\text{Lecture-II}}$

5) $\frac{1-\sin A}{1+\sin A} = (\sec A - \tan A)^2$

Pf: R.H.S $= (\sec A - \tan A)^2 = \left(\frac{1}{\cos A} - \frac{\sin A}{\cos A}\right)^2 \quad \left\{\because \sec A = \frac{1}{\cos A} \text{ \& } \tan A = \frac{\sin A}{\cos A}\right\}$

$$= \left(\frac{1-\sin A}{\cos A}\right)^2 = \frac{(1-\sin A)^2}{\cos^2 A}$$

$$= \frac{(1-\sin A)^2}{1-\sin^2 A} \quad \left\{\because \cos^2 A = 1-\sin^2 A\right\}$$

$$= \frac{(1-\sin A)^{\cancel{2}}}{\cancel{(1-\sin A)}(1+\sin A)} = \frac{1-\sin A}{1+\sin A} = \text{L.H.S}$$

H.P.

(M-II) $L.H.S = \frac{1-\sin A}{(1+\sin A)}\frac{(1-\sin A)}{(1-\sin A)} = \frac{(1-\sin A)^2}{(1-\sin^2 A)}$

$$= \frac{(1-\sin A)^2}{\cos^2 A} \quad \left\{ \because 1-\sin^2 A = \cos^2 A \right\}$$

$$= \left(\frac{1-\sin A}{\cos A}\right)^2 = \left(\frac{1}{\cos A} - \frac{\sin A}{\cos A}\right)^2$$

$$= (\sec A - \tan A)^2 = R.H.S. \quad \left\{ \begin{array}{l} \because \frac{1}{\cos A} = \sec A \\ \& \frac{\sin A}{\cos A} = \tan A \end{array} \right\}.$$

6) $\frac{\tan^2\theta}{(\sec\theta-1)^2} = \frac{1+\cos\theta}{1-\cos\theta}$

Pf: $L.H.S = \frac{\tan^2\theta}{(\sec\theta-1)^2} = \frac{(\sec^2\theta-1)}{(\sec\theta-1)^2} \quad \left\{ \because \tan^2\theta = \sec^2\theta - 1 \right\}$

$$= \frac{\cancel{(\sec\theta-1)}(\sec\theta+1)}{(\sec\theta-1)^{\cancel{2}}} = \frac{\sec\theta+1}{\sec\theta-1} = \frac{\frac{1}{\cos\theta}+1}{\frac{1}{\cos\theta}-1}$$

$$= \frac{\frac{1+\cos\theta}{\cancel{\cos\theta}}}{\frac{1-\cos\theta}{\cancel{\cos\theta}}} = \frac{1+\cos\theta}{1-\cos\theta} = R.H.S. \quad \left\{ \because \sec\theta = \frac{1}{\cos\theta} \right\}$$

7) $\sqrt{\dfrac{1-\cos A}{1+\cos A}} = \dfrac{\sin A}{1+\cos A}$.

Pf:- $L.H.S = \sqrt{\dfrac{(1-\cos A)}{(1+\cos A)}\dfrac{(1+\cos A)}{(1+\cos A)}} = \sqrt{\dfrac{1-\cos^2 A}{(1+\cos A)^2}}$

$= \sqrt{\dfrac{\sin^2 A}{(1+\cos A)^2}} \quad \left\{ \because 1-\cos^2 A = \sin^2 A \right\}$

$= \dfrac{\sin A}{1+\cos A} = R.H.S.$

8) $\sqrt{\dfrac{1+\sin A}{1-\sin A}} = \dfrac{\cos A}{1-\sin A}$.

Imp.

P.T

9) $\dfrac{\tan A + \sec A - 1}{\tan A - \sec A + 1} = \dfrac{1+\sin A}{\cos A}$.

Pf:- $L.H.S = \dfrac{(\tan A + \sec A) - (\sec^2 A - \tan^2 A)}{(\tan A - \sec A + 1)} \quad \left\{ \because 1 = \sec^2 A - \tan^2 A \right\}$

$= \dfrac{(\tan A + \sec A) - (\sec A + \tan A)(\sec A - \tan A)}{(\tan A - \sec A + 1)}$

$= \dfrac{(\sec A + \tan A)(1 - (\sec A - \tan A))}{(\tan A - \sec A + 1)}$

$$\frac{\tan A+\sec A-1}{\tan A-\sec A+1}=\frac{1+\sin A}{\cos A}.$$

$$1=\sec^2A-\tan^2A$$

Pf: $$L\cdot H\cdot S=\frac{(\tan A+\sec A)-(\sec^2A-\tan^2A)}{\tan A-\sec A+1}$$

$$=\frac{(\tan A+\sec A)-(\sec A-\tan A)(\sec A+\tan A)}{\tan A-\sec A+1}$$

$$=\frac{(\sec A+\tan A)\left[1-(\sec A-\tan A)\right]}{(\tan A-\sec A+1)}$$

$$=\frac{(\sec A+\tan A)\cancel{(1-\sec A+\tan A)}}{\cancel{(\tan A-\sec A+1)}}=\sec A+\tan A$$

$$=\frac{1}{\cos A}+\frac{\sin A}{\cos A}=\frac{1+\sin A}{\cos A}=R\cdot H\cdot S.$$

$$= \frac{(\sec A+\tan A)\cancel{(1-\sec A+\tan A)}}{\cancel{(\tan A-\sec A+1)}} = \sec A + \tan A$$

$$= \frac{1}{\cos A} + \frac{\sin A}{\cos A} = \frac{1+\sin A}{\cos A} = R.H.S.$$

10) $$\frac{\cot A + \operatorname{cosec} A - 1}{\cot A - \operatorname{cosec} A + 1} = \frac{1+\cos A}{\sin A}.$$

Pf: $$L.H.S = \frac{(\cot A + \operatorname{cosec} A) - (\operatorname{cosec}^2 A - \cot^2 A)}{(\cot A - \operatorname{cosec} A + 1)} \quad \left\{ \because \operatorname{cosec}^2 A - \cot^2 A = 1 \right\}.$$

$$= \frac{(\cot A + \operatorname{cosec} A) - (\operatorname{cosec} A + \cot A)(\operatorname{cosec} A - \cot A)}{(\cot A - \operatorname{cosec} A + 1)}$$

$$= \frac{(\operatorname{cosec} A + \cot A)[1 - (\operatorname{cosec} A - \cot A)]}{(\cot A - \operatorname{cosec} A + 1)}$$

$$= \frac{(\operatorname{cosec} A + \cot A)\cancel{[1 - \operatorname{cosec} A + \cot A]}}{\cancel{(\cot A - \operatorname{cosec} A + 1)}} = \operatorname{cosec} A + \cot A$$

$$= \frac{1}{\sin A} + \frac{\cos A}{\sin A} = \frac{1+\cos A}{\sin A} = R.H.S.$$

11) $$\frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A.$$ Sol: $$L.H.S = \frac{\sin A(1-2\sin^2 A)}{\cos A(2\cos^2 A - 1)}$$

$$= \tan A\left[\frac{1-2(1-\cos^2 A)}{2\cos^2 A - 1}\right] = \tan A\left(\frac{\cancel{1-2+2\cos^2 A}}{\cancel{2\cos^2 A - 1}}\right) = \tan A = R.H.S$$

H.P.

Lecture-III

$(a+b)^2 = a^2+b^2+2ab \longrightarrow a^2+b^2 = (a+b)^2-2ab$

12) $\sin^4 A + \cos^4 A = 1-2\sin^2 A\cos^2 A.$

Pf:- $L.H.S = (\sin^2 A)^2 + (\cos^2 A)^2 = (\sin^2 A+\cos^2 A)^2 - 2\sin^2 A\cos^2 A$

$\{\because a^2+b^2 = (a+b)^2-2ab$
where; $a=\sin^2 A$ & $b=\cos^2 A\}.$

$\{\because \sin^2 A+\cos^2 A = 1\}.$

$= (1)^2 - 2\sin^2 A\cos^2 A = 1-2\sin^2 A\cos^2 A = R.H.S.$

H.P.

$(a+b)^3 = a^3+b^3+3ab(a+b) \longrightarrow a^3+b^3 = (a+b)^3-3ab(a+b)$

13) $\sin^6 A+\cos^6 A = 1-3\sin^2 A\cos^2 A.$ $\{a^6 = (a^2)^3\}$

Pf:- $L.H.S = (\sin^2 A)^3 + (\cos^2 A)^3 = (\sin^2 A+\cos^2 A)^3 - 3\sin^2 A\cos^2 A(\sin^2 A+\cos^2 A)$

$= (1)^3 - 3\sin^2 A\cos^2 A(1)$ $\{\because \sin^2 A+\cos^2 A=1\}$

$= 1-3\sin^2 A\cos^2 A = R.H.S$

H.P.

14) $\sec^4 A - \sec^2 A = \tan^2 A + \tan^4 A.$

Pf:- $L.H.S = \sec^2 A(\sec^2 A-1) = (\sec^2 A)(\tan^2 A)$ $\{\because \sec^2 A - 1 = \tan^2 A\}$

$= (1+\tan^2 A)(\tan^2 A)$ $\{\because 1+\tan^2 A = \sec^2 A\}$

$= \tan^2 A + \tan^4 A = R.H.S.$

H.P.

15) $\dfrac{(\sin\theta)(\tan\theta)}{1-\cos\theta} = 1+\sec\theta$

Pf: $L.H.S = \dfrac{(\sin\theta)(\sin\theta)}{\cos\theta\,(1-\cos\theta)}$ $\left\{ \because \tan\theta = \dfrac{\sin\theta}{\cos\theta} \right\}$

$$= \frac{\sin^2\theta}{\cos\theta\,(1-\cos\theta)} = \frac{1-\cos^2\theta}{\cos\theta\,(1-\cos\theta)} = \frac{\cancel{(1-\cos\theta)}(1+\cos\theta)}{\cos\theta\,\cancel{(1-\cos\theta)}}$$

$\left\{ \sin^2\theta = 1-\cos^2\theta \right\}$

$$= \frac{1+\cos\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta} + \frac{\cos\theta}{\cos\theta} = \sec\theta + 1 = R.H.S.$$

4. $\dfrac{1}{(\tan A + \cot A)} = \cos A \sin A$

$$\text{'H.S} = \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}} = \frac{1}{\frac{\sin^2 A + \cos^2 A}{\cos A \sin A}} = \frac{\cos A \sin A}{\sin^2 A + \cos^2 A}$$

$$= \frac{\cos A \sin A}{1} \quad \left\{ \because \sin^2 A + \cos^2 A = 1 \right\}$$

$$= \cos A \sin A = \text{R.H.S.}$$

H.P.

6. $(1 + \cot A)^2 + (1 - \cot A)^2 = 2 \operatorname{cosec}^2 A$

$$\text{L.H.S} = 1 + \cot^2 A + \cancel{2\cot A} + 1 + \cot^2 A - \cancel{2\cot A}$$

$$= \operatorname{cosec}^2 A + \operatorname{cosec}^2 A$$

$$= 2\operatorname{cosec}^2 A = \text{R.H.S.}$$

sin A (1 + tan A) + cos A (1 + cot A) = sec A + cosec A

L.H.S =

Sol:- $\sin A\left(1+\frac{\sin A}{\cos A}\right)+\cos A\left(1+\frac{\cos A}{\sin A}\right)$

$$= \sin A\left(\frac{\cos A+\sin A}{\cos A}\right)+\cos A\left(\frac{\sin A+\cos A}{\sin A}\right) = (\sin A+\cos A)\left(\frac{\sin A}{\cos A}+\frac{\cos A}{\sin A}\right)$$

$$= (\sin A+\cos A)\left(\frac{\sin^2 A+\cos^2 A}{\cos A\sin A}\right) = \frac{(\sin A+\cos A)}{\cos A\sin A} = \frac{\cancel{\sin A}}{\cos A\cancel{\sin A}}+\frac{\cancel{\cos A}}{\cancel{\cos A}\sin A}$$

$$= \sec A + \operatorname{cosec} A = R.H.S.$$

20. $\dfrac{\operatorname{cosec} A}{(\operatorname{cosec} A - 1)} + \dfrac{\operatorname{cosec} A}{(\operatorname{cosec} A + 1)} = 2\sec^2 A$

Pf:- $L.H.S = \operatorname{Cosec}A\left[\dfrac{1}{\operatorname{Cosec}A - 1} + \dfrac{1}{\operatorname{Cosec}A + 1}\right] = \operatorname{Cosec}A\left[\dfrac{\operatorname{Cosec}A + \cancel{1} + \operatorname{Cosec}A - \cancel{1}}{(\operatorname{Cosec}A - 1)(\operatorname{Cosec}A + 1)}\right]$

$= \dfrac{2\operatorname{Cosec}^2 A}{\operatorname{Cosec}^2 A - 1} = \dfrac{2\operatorname{Cosec}^2 A}{\cot^2 A} = \dfrac{2}{\cancel{\sin^2 A} \cdot \dfrac{\cos^2 A}{\cancel{\sin^2 A}}} = 2\sec^2 A = R.H.S.$

$\left\{ \because \operatorname{Cosec}^2 A - 1 = \cot^2 A;\ \operatorname{Cosec}A = \dfrac{1}{\sin A},\ \cot A = \dfrac{\cos A}{\sin A} \right\}$

21. $(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) = 2$

Pf:- $L.H.S = \left(1 + \dfrac{\cos A}{\sin A} - \dfrac{1}{\sin A}\right)\left(1 + \dfrac{\sin A}{\cos A} + \dfrac{1}{\cos A}\right) = \dfrac{[(\sin A + \cos A) - 1]}{\sin A}\,\dfrac{[(\cos A + \sin A) + 1]}{\cos A}$

$= \dfrac{(\sin A + \cos A)^2 - 1}{\sin A \cos A} = \dfrac{\cancel{\sin^2 A + \cos^2 A} + 2\sin A\cos A \cancel{-1}}{\sin A \cos A}$

$= \dfrac{2\sin A\cos A}{\sin A \cos A} = 2 = R.H.S.$

Lecture-IV

$a^3+b^3 = (a+b)(a^2+b^2-ab)$

$a^3-b^3 = (a-b)(a^2+b^2+ab)$.

23. $$\frac{\cos^3 A+\sin^3 A}{\cos A+\sin A}+\frac{\cos^3 A-\sin^3 A}{\cos A-\sin A}=2$$

Pf:- $L.H.S = \frac{\cancel{(\cos A+\sin A)}(\cos^2 A+\sin^2 A-\cos A\sin A)}{\cancel{(\cos A+\sin A)}}$

$+ \frac{\cancel{(\cos A-\sin A)}(\cos^2 A+\sin^2 A+\cos A\sin A)}{\cancel{(\cos A-\sin A)}}$

$= 1-\cancel{\cos A\sin A}+1+\cancel{\cos A\sin A}$ $\{\because \sin^2 A+\cos^2 A=1\}$

$= 2$

27. $$\frac{\cot A-1}{2-\sec^2 A}=\frac{\cot A}{1+\tan A}$$

Pf:- $L.H.S = \frac{1/\tan A - 1}{2-(1+\tan^2 A)}$ $\{\because \cot A = 1/\tan A$ & $\sec^2 A = 1+\tan^2 A\}$

$= \frac{(1-\tan A)/\tan A}{1-\tan^2 A} = \frac{\cancel{(1-\tan A)}}{(\tan A)\cancel{(1-\tan A)}(1+\tan A)}$

$= \frac{\cot A}{1+\tan A} = R.H.S.$

28. $\dfrac{1}{(\sec A+\tan A)}-\dfrac{1}{\cos A}=\dfrac{1}{\cos A}-\dfrac{1}{(\sec A-\tan A)}$

M-I

Pf: $L.H.S. = (\sec A-\tan A)-\sec A$

$= -\tan A$

$\because \sec^2 A-\tan^2 A=1$

$\Rightarrow \sec A-\tan A=\dfrac{1}{\sec A+\tan A}$

$\Rightarrow \sec A+\tan A=\dfrac{1}{\sec A-\tan A}$

$= \sec A-\tan A-\sec A$

$= \sec A-(\sec A+\tan A)$

$= \dfrac{1}{\cos A}-\dfrac{1}{\sec A-\tan A} = R.H.S.$

H.P.

M-II $L.H.S. = (\sec A-\tan A)-\sec A$

$= -\tan A$

$R.H.S = \sec A-(\sec A+\tan A) = -\tan A.$

L.H.S.=R.H.S.

H.P.

35. $\dfrac{\cos A}{1+\sin A}+\tan A=\sec A$

$L.H.S = \dfrac{\cos A}{1+\sin A}+\dfrac{\sin A}{\cos A} = \dfrac{\cos^2 A+\sin A(1+\sin A)}{(1+\sin A)(\cos A)}$

$= \dfrac{\cos^2 A+\sin A+\sin^2 A}{(1+\sin A)(\cos A)} = \dfrac{\cancel{1+\sin A}}{\cancel{(1+\sin A)}\cos A} = \sec A. = R.H.S$

H.P.

24. $\dfrac{\tan A}{(1-\cot A)} + \dfrac{\cot A}{(1-\tan A)} = \sec A \operatorname{cosec} A + 1$

Sol^n:- L.H.S =

$$\frac{\sin A}{\cos A\left(1-\frac{\cos A}{\sin A}\right)} + \frac{\cos A}{\sin A\left(1-\frac{\sin A}{\cos A}\right)} = \frac{\sin^2 A}{\cos A(\sin A-\cos A)} + \frac{\cos^2 A}{\sin A(\cos A-\sin A)}$$

$$\frac{\sin^2 A}{\cos A(\sin A-\cos A)} - \frac{\cos^2 A}{\sin A(\sin A-\cos A)} = \frac{\sin^3 A-\cos^3 A}{\cos A\sin A(\sin A-\cos A)}$$

$$\frac{\cancel{(\sin A-\cos A)}(\sin^2 A+\cos^2 A+\sin A\cos A)}{\cos A\sin A\cancel{(\sin A-\cos A)}} = \frac{1+\sin A\cos A}{\sin A\cos A}$$

$$= \frac{1}{\sin A\cos A} + \frac{\sin A\cos A}{\sin A\cos A} = \sec A\cdot\operatorname{cosec} A + 1 = R.H.S.$$

H.P

25. $\dfrac{\tan\theta+\sin\theta}{\tan\theta-\sin\theta} = \dfrac{\sec\theta+1}{\sec\theta-1}$

$$L.H.S = \frac{\frac{\sin\theta}{\cos\theta}+\sin\theta}{\frac{\sin\theta}{\cos\theta}-\sin\theta} = \frac{\cancel{\sin\theta}\left(\frac{1}{\cos\theta}+1\right)}{\cancel{\sin\theta}\left(\frac{1}{\cos\theta}-1\right)} = \frac{\sec\theta+1}{\sec\theta-1} = R.H.S$$

H.P.

29. $\dfrac{\sin A}{(\cot A + \operatorname{cosec} A)} = 2 + \dfrac{\sin A}{(\cot A - \operatorname{cosec} A)}$

M-II

Sol: $L.H.S = \dfrac{\sin A}{\dfrac{\cos A}{\sin A} + \dfrac{1}{\sin A}} = \dfrac{\sin^2 A}{\cos A + 1} = \dfrac{2(\cos A+1) + \sin^2 A - 2(\cos A+1)}{\cos A + 1}$

$= \dfrac{2(\cos A+1)}{(\cos A+1)} + \dfrac{(1-\cos^2 A) - 2(\cos A+1)}{\cos A+1}$

$= 2 + \dfrac{\cancel{(1+\cos A)}(1-\cos A-2)}{\cancel{(\cos A+1)}} = 2 + \dfrac{(-1-\cos A)(\cot A - \operatorname{cosec} A)}{(\cot A - \operatorname{cosec} A)}$

$= 2 - \dfrac{(1+\cos A)(\cos A - 1)}{\sin A(\cot A - \operatorname{cosec} A)} = 2 + \dfrac{(1-\cos^2 A)\sin A}{\sin A(\cot A - \operatorname{cosec} A)}$

$= R.H.S.$

M-I

$L.H.S = \dfrac{\sin A}{\dfrac{\cos A}{\sin A} + \dfrac{1}{\sin A}} = \dfrac{\sin^2 A}{\cos A+1} = \dfrac{1-\cos^2 A}{1+\cos A} = 1-\cos A.$

$R.H.S = 2 + \dfrac{\sin A}{\dfrac{\cos A}{\sin A} - \dfrac{1}{\sin A}} = 2 + \dfrac{\sin^2 A}{\cos A - 1} = 2 + \dfrac{\cancel{1-\cos^2 A}}{\cancel{(\cos A-1)}}$ $-(1+\cos A)$

$= 1-\cos A$

$2 + \dfrac{1-\cos^2 A}{\cos A - 1} = 2 + \dfrac{\cancel{(1-\cos A)}(1+\cos A)}{\cancel{(\cos A-1)}}$ (-1)

$= 2 - (1+\cos A) = 2 - 1 - \cos A$

$= 1 - \cos A.$

33. $\dfrac{\cot^2 A}{(\operatorname{cosec} A+1)^2}=\dfrac{1-\sin A}{1+\sin A}$

$$L.H.S = \frac{\operatorname{cosec}^2 A-1}{(\operatorname{cosec}A+1)^2} = \frac{(\operatorname{cosec}A-1)\cancel{(\operatorname{cosec}A+1)}}{(\operatorname{cosec}A+1)^{\cancel{2}}} = \frac{\operatorname{cosec}A-1}{\operatorname{cosec}A+1}$$

$$= \frac{\frac{1}{\sin A}-1}{\frac{1}{\sin A}+1} = \frac{(1-\sin A)/\cancel{\sin A}}{(1+\sin A)/\cancel{\sin A}} = \frac{1-\sin A}{1+\sin A} = R.H.S.$$

H.P.

$$\left(\frac{1-\tan\theta}{1-\cot\theta}\right)^2 = \tan^2\theta$$

Pf:- $L.H.S = \left(\dfrac{1-\tan\theta}{1-\frac{1}{\tan\theta}}\right)^2 = \left(\dfrac{\cancel{1-\tan\theta}}{\frac{\cancel{\tan\theta-1}}{\tan\theta}}\right)^2 = (-\tan\theta)^2 = \tan^2\theta$

$= R.H.S.$