Section A

1.

(c) 512

Explanation:

As order of 3×3 matrix contains 9 elements. Each element can be selected in 2 ways (it can be either 0 or 1).

Hence, all the nine netries can be chosen in $2^9 = 512$ ways (by the multiplication principle)

2. **(a)** (adj B) (adj A)

Explanation:

We Know that $(AB)^{-1} = \frac{adj(AB)}{|AB|}$

adj (AB) = $(AB)^{-1}$.|AB|

We also know that $(AB)^{-1} = B^{-1} A^{-1}$

|AB| = |A|.|B|

Putting them in (i)

adj (AB) = $B^{-1}A^{-1}$.|A|.|B|

adj (AB) = $(B^{-1},|B|)(A^{-1}, |A|)$

adj (AB) = (adjB).(adjA)

3. **(a)** hc - fg

Explanation:

$A_{21} = (-1)^{2+1} M_{21} = -M_{21} \ = -\begin{vmatrix} h & g \\ f & c \end{vmatrix}$

= -(hc - fg) = fg - hc

4.

(b) f(x) is continuous and differentiable $\forall x \in \mathbb{R}$ - {0}

Explanation:

f(x) is continuous and differentiable $\forall x \in \mathbb{R}$ - {0}

5.

(b) $\cos^{-1}\left(\frac{2}{3}\right)$

Explanation:

Direction ratios of the lines are given $2\hat{\imath} + 2\hat{\jmath} + \hat{k}$ and $4\hat{\imath} + \hat{\jmath} + 8\hat{k}$

$\vec{a} = 2\hat{i} + 2\hat{j} + \hat{k}$

$|\vec{a}| = \sqrt{2^2 + 2^2 + 1}$

$|\vec{a}| = 3$

$\vec{\mathrm{b}} = 4\hat{\mathrm{i}} + \hat{\mathrm{j}} + 8\hat{\mathrm{k}}$

$|\vec{\mathrm{b}}| = \sqrt{4^2 + 1 + 8^2}$

= 9

$\cos\alpha = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\times|\vec{b}|}$

$\cos\alpha = \frac{(2\hat{\imath}+2\hat{\jmath}+\hat{\mathrm{k}})\cdot(4\hat{\mathrm{i}}+\hat{\mathrm{j}}+8\hat{\mathrm{k}})}{3\times9}$

$\cos\alpha = \frac{8+8+2}{27}$

$\cos\alpha = \frac{2}{3}$

6. **(a)** 2, degree not defined

Explanation:

2, degree not defined

7. **(a)** 2500

Explanation:

Here , Maximize Z = 50x+60y , subject to constraints x +2 y ≤ 50 , x + y ≥ 30, x, y ≥ 0.

Corner points	Z = 50x +60 y
P(50 ,0)	2500
Q(0 , 30)	1800
R(10, 20)	1700

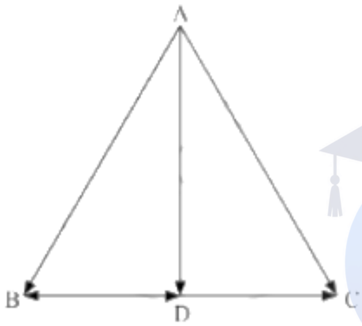
Hence, the maximum value is 2500

8.

(b) $\frac{\sqrt{34}}{2}$

Explanation:

In $\triangle ABC$,



Using the triangle law of vector addition, we have

$\overrightarrow{BC} = \overrightarrow{AC} - \overrightarrow{AB}$

$= (3\hat{i} - \hat{j} + 4\hat{k}) - (\hat{j} + \hat{k})$

$\therefore \overrightarrow{BD} = \frac{1}{2}\overrightarrow{BC} = \frac{3}{2}\hat{i} - \hat{j} + \frac{3}{2}\hat{k}$ (since AD is the median)

In $\triangle$ABD, using the triangle law of vector addition, we have

$\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BD}$

$= (\hat{j} + \hat{k}) + \left(\frac{3}{2}\hat{i} - \hat{j} + \frac{3}{2}\hat{k}\right)$

$= \frac{3}{2}\hat{i} + 0\hat{j} + \frac{5}{2}\hat{k}$

$\therefore AD = \sqrt{\left(\frac{3}{2}\right)^2 + 0^2 + \left(\frac{5}{2}\right)^2} = \frac{1}{2}\sqrt{34}$

Hence, the length of the median through A is $\frac{1}{2}\sqrt{34}$ units.

9. **(a)** 2

Explanation:

cos x is an even function so,

$\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$

$\therefore \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x dx = 2\int_{0}^{\frac{\pi}{2}} \cos x dx$

= 2 (1 - 0)

= 2

10. **(a)** skew-symmetric matrix

Explanation:

We have matrices A and B of same order.

Let P = (AB' − BA')

Then, P′ = (AB′ − BA′)'

= (AB′)′ − (BA′)′

= (B′)′ (A)′ − (A′)′B′ = BA′ − AB′ = −(AB′ − BA′) = −P

Therefore, the given matrix (AB - BA') is a skew-symmetric matrix.

11.

(c) no feasible solution

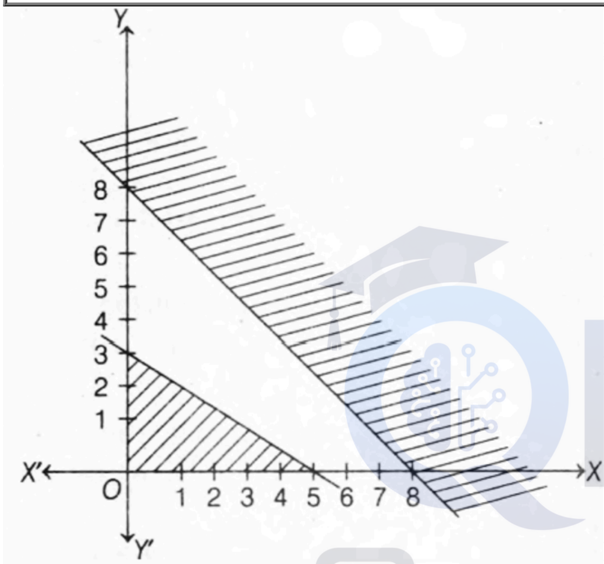
Explanation:

Table for equation x + y = 8 is

x	0	8
y = 8 - x	8	0

Table for equation 3x + 5y = 15 is

x	0	5
y = $\frac{15-3x}{5}$	3	0



It can be concluded from the graph, that there is no point, which can satisfy all the constraints simultaneously. Therefore, the problem has no feasible solution.

12.

(c) -2

Explanation:

Given $\vec{a} = 2\hat{i} + 4\hat{j} - \hat{k}\ and\ \vec{b} = 3\hat{i} - 2\hat{j} + \lambda\hat{k}$ such that $\vec{a} \perp \vec{b}$

$\therefore$ 6 - 8 - $\lambda = 0$ = -2 - $\lambda = 0$

$\Rightarrow \lambda = -2$

13.

(b) $\frac{33}{2}$

Explanation:

When a given matrix is singular then the given matrix determinant is 0.

|A| = 0

Given,

$$A = \begin{pmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 3 & -4 \end{pmatrix}$$

|A| = 0

1(-4k + 6) -k(-12 + 4) + 3(9 - 2k) = 0

-4k + 6 + 12k - 4k + 27 - 6k = 0

-2k + 33 = 0

k = $\frac{33}{2}$.

Which is the required solution.

14.

(c) $P(A \cap B) = \frac{1}{2}P(B)$

Explanation:

$P(A \cap B) = \frac{1}{2}P(B)$

15.

(d) $x = \ \nu y$

Explanation:

A homogeneous equation of the form $\frac{dy}{dx} = h\left(\frac{x}{y}\right)$ can be solved by making the substitution x= vy.so that it becomes variable separable form and integration is then possible

16. **(a)** $6\sqrt{3}$

Explanation:

$|\vec{a}| = 3|\vec{b}| = 4$ and $|\vec{a} \times \vec{b}| = 6$

$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta.$

$6 = 3 \times 4 \sin\theta.$

$\frac{bx\cancel{2}}{3\times 4_2} = \sin\theta.$

$\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}$

$\vec{a} \cdot \bar{b} = |\vec{a}| \cdot |\vec{b}| \cos\theta$

$= 3 \times 4 \times \cos\frac{\pi}{6}$

$= 3 \times 4 \times \frac{\sqrt{3}}{2}$

$= 6\sqrt{3}.$

17.

(d) $\frac{-\sqrt{y}}{\sqrt{x}}$

Explanation:

Given that $\sqrt{x} + \sqrt{y} = \sqrt{a}$

Differentiating with respect to x, we obtain

$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0$

Or $\frac{dy}{dx} = -\sqrt{\frac{y}{x}}$

18.

(d) $\frac{\pi}{2}$

Explanation:

Let's consider the first parallel vector to be $\vec{a} = a\hat{i} + b\hat{j} + c\hat{k}$ and second parallel vector be

$\overrightarrow{\mathrm{b}} = (\mathrm{b} - \mathrm{c})\hat{\mathrm{i}} + (\mathrm{c} - \mathrm{a})\hat{\mathrm{j}} + (\mathrm{a} - \mathrm{b})\hat{\mathrm{k}}$

For the angle, we can use the formula $\cos\alpha = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\times|\vec{b}|}$

For that, we need to find the magnitude of these vectors

$|\vec{a}| = \sqrt{a^2 + b^2 + (c)^2}$

$= \sqrt{a^2 + b^2 + c^2}$

$|\overrightarrow{\mathrm{b}}| = \sqrt{(\mathrm{b} - \mathrm{c})^2 + (\mathrm{c} - \mathrm{a})^2 + (\mathrm{a} - \mathrm{b})^2}$

$= \sqrt{2\left(a^2 + b^2 + c^2 - ab - bc - ca\right)}$

$\Rightarrow \cos\alpha = \frac{(a\hat{i}+b\hat{j}+c\hat{k})\cdot((b-c)\hat{i}+(c-a)\hat{j}+(a-b)\hat{k})}{\sqrt{2\left(a^2+b^2+c^2-ab-bc-ca\right)}\times\sqrt{a^2+b^2+c^2}}$

$$\Rightarrow \cos\alpha = \frac{ab-ac+bc-ba+ca-cb}{\sqrt{2\left(a^2+b^2+c^2-ab-bc-ca\right)}\times\sqrt{a^2+b^2+c^2}}$$
$$\Rightarrow \cos\alpha = \frac{0}{\sqrt{2\left(a^2+b^2+c^2-ab-bc-ca\right)}\times\sqrt{a^2+b^2+c^2}}$$
$$\Rightarrow \alpha = \cos^{-1}(0)$$
$$\therefore \alpha = \frac{\pi}{2}$$

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Both A and R are true and R is the correct explanation of A.

20.

(d) A is false but R is true.

Explanation:

R = {(1, 3), (4, 2) (2, 7) (2, 3) (3, 1)}

As (2, 3) $\in$ R but (3, 2) $\notin$ R

So, set 'A' is not symmetric.

Section B

21. Let $\cot^{-1}\left(\frac{-5}{12}\right) = y$

Then $\cot y = \frac{-5}{12}$

Now,

$\sin\left[2\cot^{-1}\left(\frac{-5}{12}\right)\right] = \sin 2y$

$= 2\sin y\cos y = 2\left(\frac{12}{13}\right)\left(\frac{-5}{13}\right)$ $\left[\text{since } \cot y < 0, \text{ so } y \in \left(\frac{\pi}{2}, \pi\right)\right]$

$= \frac{-120}{169}$

OR

We have to find principle value of , $\cos^{-1}\left(\frac{1}{2}\right)$

Lets $\cos^{-1}\frac{1}{2} = \theta$

$\Rightarrow \frac{1}{2} = \cos\theta$

$\Rightarrow \cos\frac{\pi}{3} = \cos\theta$

$\Rightarrow \theta = \frac{\pi}{3}$

22. We have Max. value is $\frac{3}{4}$ at $x = \frac{\pi}{6}$ and min. value is $\frac{1}{2}$ at $x = \frac{\pi}{2}$

Also F'(x) = cos x - $\frac{1}{2}$sin x = 0

2 cos x = sin x

$\frac{\pi}{6} = \frac{\pi}{3}$

$f\left(\frac{\pi}{2}\right) = \sin\frac{\pi}{2} + \frac{1}{2}\cos\frac{\pi}{2} = \frac{1}{2}$

$f\left(\frac{\pi}{6}\right) = \sin\frac{\pi}{6} + \frac{1}{2}\cos\frac{\pi}{6} = \frac{1}{2} + \frac{\sqrt{3}}{4}$

$f\left(\frac{\pi}{3}\right) = \sin\frac{\pi}{3} + \frac{1}{2}\cos\frac{\pi}{3} = \frac{\sqrt{3}}{2} + \frac{1}{4}$

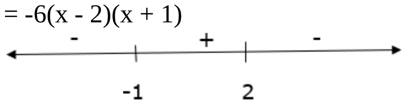
23. The given function is,

f (x) = -2x^3 + 3x^2 + 12x + 6

f'(x) = -6x^2 + 6x + 12

f'(x) = -6(x^2 - x - 2)

= -6(x - 2)(x + 1)



Function f(x) is increasing for x $\in$ [-1, 2] and decreasing in x $\in$ (-∞, -1) $\cup$ (2, ∞).

OR

Here,

$\frac{dx}{dt} = 4$ units / sec and $x = 2$

And, $y = 7x - x^3$

Slope of the curve (S) $= \frac{dy}{dx}$

$S = \frac{dy}{dx} = 7 - 3x^2$

$\frac{ds}{dt} = -6x \times \frac{dx}{dt}$

$= -6(2)(4)$

= - 48 units/ sec

So , slope is decreasing at the rate of 48 units/ sec

24. For the given integral is

Let $x = a\sin\theta$. Then, $dx = d(a\sin\theta) = a\cos\theta d\theta$

Also, $x = 0 \Rightarrow a\sin\theta = 0 \Rightarrow \sin\theta = 0 \Rightarrow \theta = 0$

And, $x = a \Rightarrow a\sin\theta = a \Rightarrow \sin\theta = 1 \Rightarrow \theta = \frac{\pi}{2}$

$\therefore \quad I = \int_0^a \frac{x^4}{\sqrt{a^2-x^2}}dx = \int_0^{\pi/2} \frac{(a\sin\theta)^4}{\sqrt{a^2-a^2\sin^2\theta}} a\cos\theta d\theta = a^4\int_0^{\pi/2}\sin^4\theta d\theta$

$\Rightarrow \quad I = a^4 \times \frac{3\pi}{16} = \frac{3\pi a^4}{16}$

25. Given: f(x) = (x - 1) (x + 2)^2

$\because$ f'(x) = (x + 2) + 2(x - 1) (x + 2)

= (x + 2) (x + 2 + 2x - 2)

= (x + 2) (3x)

To find the point of maxima and minima we must have

f'(x) = 0

$\Rightarrow$ (x + 2) $\times$ 3x = 0

$\Rightarrow$ x = 0, - 2

At x = -2 f'(x) changes from + ve to - ve

$\because$ x = -2 is point of local maxima

At x = 0 f'(x) changes from - ve to -ve

$\because$ x = 0 is point of local minima

Thus, local min value = f(0) = - 4

local max value f(-2) = 0.

Section C

26. Let I = $\int \frac{x^2(x^4+4)}{x^2+4}dx$

$= \int \left(\frac{x^6+4x^2}{x^2+4}\right)dx$

Therefore by long division we have

$$\begin{array}{r|l} & x^4 - 4x^2 + 20 \\ \hline x^2+4 & x^6 + 4x^2 \\ & x^6 + 4x^4 \\ & - \quad - \\ \hline & -4x^4 + 4x^2 \\ & -4x^4 - 16x^2 \\ & + \quad + \\ \hline & 20x^2 \\ & 20x^2 + 80 \\ & - \quad - \\ \hline & -80 \\ \hline \end{array}$$

Therefore,

$\frac{x^2(x^4+4)}{(x^2+4)}$ = (x^4 - 4x^2 + 20) - $\frac{80}{x^2+4}$

I = $\int \frac{x^2(x^4+4)}{(x^2+4)}dx$

= $\int$ (x^4 - 4x^2 + 20) dx - 80 $\int \frac{dx}{x^2+2^2}$

= $\int$ x^{4}dx - 4 $\int$ x^{2}dx + 20 $\int$ dx - 80 $\int \frac{dx}{x^2+2^2}$

$= \frac{x^{4+1}}{4+1} - 4\left[\frac{x^3}{3}\right]$ + 20(x) - 80 $\times \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$

$= \frac{x^5}{5} - \frac{4}{3}x^3$ + 20x - 40 tan^{-1} $\left(\frac{x}{2}\right)$ + C

27. Let p be the probability of getting 1 white ball out of 7 red, 5 white and 8 black balls. Therefore,we have,

$p = \frac{5}{20}$

$q = 1 - \frac{1}{4}$

$p = \frac{1}{4}$ [since p + q = 1]

$q = \frac{3}{4}$

Let X be a random variable denoting the number of white balls selected with replacement out of 4 balls. Then,

Probability of getting r white balls out of n balls is given by

P(X=r) = ${}^nC_r p^r q^{n-r}$

Probability of getting all white balls

= P(X = 4)

$= {}^4C_4\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right)^{4-4}$

$= \left(\frac{1}{4}\right)^4$

28. According to the question, $I = \int \frac{\cos\theta}{(4+\sin^2\theta)(5-4\cos^2\theta)]}d\theta$

$= \int \frac{\cos\theta}{(4+\sin^2\theta)[5-4(1-\sin^2\theta)]}d\theta$ $[\because cos^2\theta = 1 - sin^2\theta]$

$= \int \frac{\cos\theta}{(4+\sin^2\theta)(5-4+4\sin^2\theta)}d\theta$

$= \int \frac{\cos\theta}{(4+\sin^2\theta)(1+4\sin^2\theta)}d\theta$

Let $\sin\theta = t \Rightarrow \cos\theta d\theta = dt$

Then, $I = \int \frac{dt}{(4+t^2)(1+4t^2)}$

let , $\frac{1}{(4+t^2)(1+4t^2)} = \frac{A}{4+t^2} + \frac{B}{1+4t^2}$

using partial fractions

At t = 0, $\frac{A}{4} + \frac{B}{1} = \frac{1}{4\times1} \Rightarrow \quad A + 4B = 1$ ...(i)

At $t = 1$, $\frac{A}{5} + \frac{B}{5} = \frac{1}{5\times5} \Rightarrow 5A + 5B = 1$ ...(ii)

On solving Equations (i) and (ii), we get

$A = \frac{-1}{15}$ and $B = \frac{4}{15}$

$\frac{1}{(4+t^2)(1+4t^2)} = \frac{-\frac{1}{15}}{4+t^2} + \frac{\frac{4}{15}}{1+4t^2}$

$\Rightarrow \quad \frac{1}{(4+t^2)(+4t^2)} = \frac{-1}{15(4+t^2)} + \frac{4}{15(1+4t^2)}$

Integrating both sides w.r.t. t,

$\Rightarrow \int \frac{1}{(4+t^2)(1+4t^2)}dt = \frac{-1}{15}\int \frac{1}{4+t^2}dt + \frac{4}{15}\int \frac{1}{1+4t^2}dt$

$= \frac{-1}{15}\int \frac{1}{2^2+t^2} + \frac{4}{15\times4}\int \frac{1}{\left(\frac{1}{2}\right)^2+t^2}dt$

$= \frac{-1}{15}\cdot\frac{1}{2}\tan^{-1}\frac{t}{2} + \frac{1}{15}\cdot\frac{1}{1/2}\tan^{-1}\frac{t}{1/2} + C$ $\left[\because \int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c\right]$

put $t = \sin\theta$

$= \frac{-1}{30}\tan^{-1}\frac{\sin\theta}{2} + \frac{2}{15}\tan^{-1}2\sin\theta + C$

OR

Let I = $\int_0^{\pi/6}\left(2+3x^2\right)\cos 3x dx$.

Then, solving by means of integration by parts, we have

$I = \left[\left(2+3x^2\right)\times\frac{1}{3}\sin 3x\right]_0^{\pi/6} - \int_0^{\pi/6} 6x \times \frac{1}{3}\sin 3x dx$

$= \left[\frac{1}{3}\left(2+3x^2\right)\sin 3x\right]_0^{\pi/6} - 2\int_0^{\pi/6} x\sin 3x dx$

$= \left[\frac{1}{3}\left(2+3x^2\right)\sin 3x\right]_0^{\pi/6} - 2\left[\left[\frac{-x\cos 3x}{3}\right]_0^{\pi/6} - \int_0^{\pi/6} -\frac{\cos 3x}{3}dx\right]$

$= \left[\frac{1}{3}\left(2+3x^2\right)\sin 3x\right]_0^{\pi/6} - 2\left[\left[\frac{-x\cos 3x}{3}\right]_c^{\pi/6} + \frac{1}{9}[\sin 3x]_0^{\pi/6}\right]$

$= \left[\frac{1}{3}\left(2+\frac{\pi^2}{12}\right)\sin\frac{\pi}{2} - \frac{1}{3}(2+0)0\right] - 2\left[\left\{\left(-\frac{\pi}{18}\cos\frac{\pi}{2}\right) + \frac{0\cos 0}{3}\right\} + \frac{1}{9}\left\{\sin\frac{\pi}{2} - \sin 0\right\}\right]$

$= \left[\frac{1}{3}\left(2+\frac{\pi^2}{12}\right) - \frac{2}{3}\times 0\right] - 2\left[(0-0)+\frac{1}{9}(1-0)\right]$

$= \frac{2}{3} + \frac{\pi^2}{36} - \frac{2}{9}$

$= \frac{\pi^2}{36} + \frac{4}{9}$

$= \frac{1}{36}\left(\pi^2 + 16\right)$

29. The given differential equation is,

$(y^2 - 2xy)\,dx = (x^2 - 2xy)\,dy$

$\frac{dy}{dx} = \frac{y^2-2xy}{x^2-2xy}$

This is a homogeneous differential equation

Putting y = vx and $\frac{dy}{dx}$ = v + x$\frac{dv}{dx}$, we have,

$v + x\frac{dv}{dx} = \frac{v^2x^2-2xvx}{x^2-2xvx}$

$v + x\frac{dv}{dx} = \frac{v^2-2v}{1-2v}$

$x\frac{dv}{dx} = \frac{v^2-2v}{1-2v} - v$

$x\frac{dv}{dx} = \frac{v^2-2v-v+2v^2}{1-2v}$

$x\frac{dv}{dx} = \frac{3v^2-3v}{1-2v}$

$\frac{1-2v}{3(v^2-v)}dv = \frac{dx}{x}$

$\frac{-(2v-1)}{3(v^2-v)}dv = \frac{dx}{x}$

$\frac{(2v-1)}{(v^2-v)}dv = -3\frac{dx}{x}$

Integrating both Sides we get,

$\int \frac{(2v-1)}{(v^2-v)}dv = -3\int\frac{dx}{x}$

$\log|v^2 - v| = -3\log|x| + \log c$

$v^2 - v = \frac{c}{x^3}$

$\frac{y^2}{x^2} - \frac{y}{x} = \frac{c}{x^3}$

$y^2 - xy = \frac{c}{x}$

$x(y^2 - xy) = c$

This is the required differential equation.

OR

The given differential equation is,

$xe^{y/x} - y\sin\left(\frac{y}{x}\right) + x\frac{dy}{dx}\sin\left(\frac{y}{x}\right) = 0$

$\Rightarrow \frac{dy}{dx} = \frac{y\sin\left(\frac{y}{x}\right)-xe^{y/x}}{x\sin\left(\frac{y}{x}\right)}$

This is a homogeneous differential equation.

Putting y = vx and $\frac{dy}{dx} = v + x\frac{dv}{dx}$, then,we have,

$v + x\frac{dv}{dx} = \frac{v\sin v-e^v}{\sin v}$

$\Rightarrow x\frac{dv}{dx} = \frac{v\sin v-e^v}{\sin v} - v$

$\Rightarrow x\frac{dv}{dx} = -\frac{e^v}{\sin v}$

$\Rightarrow e^{-v}\sin v\,dv = -\frac{dx}{x}$

$\Rightarrow \int e^{-v}\sin v\,dv = -\int\frac{1}{x}\,dx$

$\Rightarrow \frac{e^{-v}}{2}(-\sin v - \cos v) = -\log|x| + \log C$ $\left[\because \int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)\right]$

$\Rightarrow -\frac{1}{2}e^{-y/x}\left\{\sin\left(\frac{y}{x}\right)+\cos\left(\frac{y}{x}\right)\right\} = -\log|x| + \log C$

$\Rightarrow e^{-y/x}\left\{\sin\left(\frac{y}{x}\right)+\cos\left(\frac{y}{x}\right)\right\} = 2\log|x| - 2\log C$

$\Rightarrow e^{-y/x}\left\{\sin\left(\frac{y}{x}\right)+\cos\left(\frac{y}{x}\right)\right\} = \log|x|^2 - 2\log C$ ...(ii)

It is given that y(1) = 0 i.e., y = 0 when x = 1. Putting these values in (ii), we get

$1 = 0 - 2\log C \Rightarrow \log C = -\frac{1}{2}$

Putting log C = $-\frac{1}{2}$ in (ii), we get

$e^{-y/x}\left\{\sin\left(\frac{y}{x}\right)+\cos\left(\frac{y}{x}\right)\right\}$ = log $|x|^2$ + 1 as the required solution.

30. First, we will convert the given inequations into equations, we obtain the following equations:

2x + y = 18, 3x + 2y = 34

Region represented by 2x + y $\geq$ 18 :

The line 2x + y = 18 meets the coordinate axes at A(9,0) and B(0,18) respectively. By joining these points we obtain the line 2x + y = 18 Clearly (0,0) does not satisfies the inequation 2x + y $\geq$ 18 . So, the region in xy plane which does not contain the origin represents the solution set of the inequation 2x + y $\geq$ 18.

Region represented by 3x + 2y $\leq$ 34 :

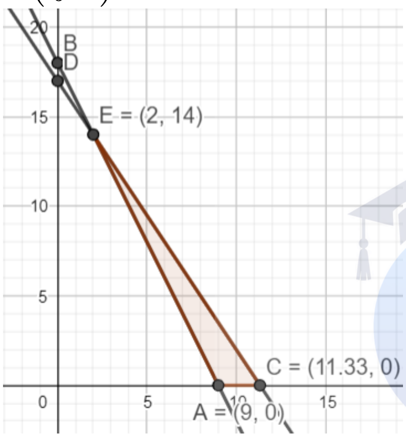
The line 3x + 2y = 34 meets the coordinate axes at

$C\left(\frac{34}{3},0\right)$ and D(0,17) respectively.

By joining these points we obtain the line 3x + 2y = 34 Clearly (0,0) satisfies the inequation 3x + 2y $\leq$ 34 . So, the region containing the origin represents the solution set of the inequation 3x + 2y $\leq$ 34

The corner points of the feasible region are A(9,0)

$C\left(\frac{34}{3},0\right)$ and E(2,14) and feasible region is bounded



The values of Z objective function at these corner points are as follows.

Corner point	Z = 50x + 30y
A(9, 0)	$50\times 9+3\times 0=450$
$C\left(\frac{34}{3},0\right)$	$50\times\frac{34}{3}+30\times 0=\frac{1700}{3}$
E(2, 14)	$50\times 2+30\times 14=520$

Therefore, the maximum value of objective function Z is

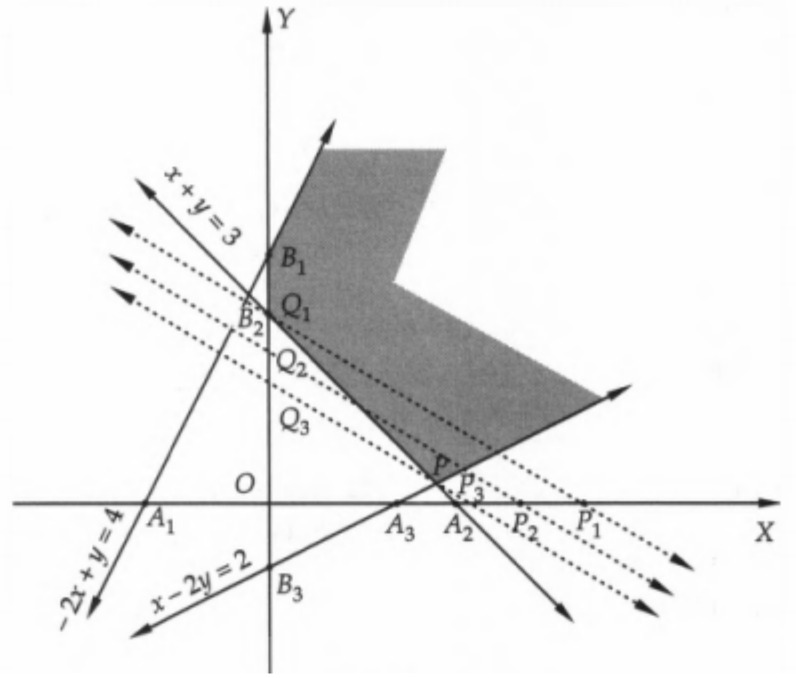
$\frac{1700}{3}$ at the point $\left(\frac{34}{3},0\right)$ Hence, x = $\frac{34}{3}$ and y = 0 is the optimal solution of the given LPP.

Thus, the optimal value of objective function Z is $\frac{1700}{3}$.

OR

Converting the inequations into equations, we obtain the lines -2x + y = 4, x + y = 3 , x - 2y = 2, x = 0 and y = 0.

These lines are drawn on a suitable scale and the feasible region of the LPP is shaded in Fig.



Now, give a value, say 15 equal to (lcm of 3 and 5) to Z to obtain the line 3x + 5y = 15. This line meets the coordinate axes at P_1 (5, 0) and Q_1(0, 3). Join these points by a dotted line. Move this line parallel to itself in the decreasing direction towards the origin so that it passes through only one point of the feasible region. Clearly, P_3Q_3 is such a line passing through the vertex P of the feasible region.

The coordinates of P are obtained by solving the lines x - 2y = 2 and x + y = 3.

Solving these equations, we get x = $\frac{8}{3}$ and y = $\frac{1}{3}$.

Put, x = $\frac{8}{3}$ and y = $\frac{1}{3}$ in Z = 3x + 5y, we get

Z = 3 × $\frac{8}{3}$ = 5 × $\frac{1}{3}$ = $\frac{29}{3}$

Hence, the minimum value of Z is $\frac{29}{3}$ at x = $\frac{8}{3}$, y = $\frac{1}{3}$.

31. Given, x = a$\left(\cos t + \log\tan\frac{t}{2}\right)$ ....(i)

and y = a sin t..........(ii)

Therefore, on differentiating both sides w.r.t t, we get,

$$\frac{dx}{dt} = a\left[\frac{d}{dt}(\cos t) + \frac{d}{dt}\log\tan\frac{t}{2}\right]$$

$$= a\left[-\sin t + \frac{1}{\tan\frac{t}{2}}\frac{d}{dt}\left(\tan\frac{t}{2}\right)\right]$$ [by using chain rule of derivative]

$$= a\left[-\sin t + \frac{1}{\tan\frac{t}{2}}\cdot\sec^2\frac{t}{2}\frac{d}{dt}\left(\frac{t}{2}\right)\right]$$

$$= a\left[-\sin t + \frac{1}{\tan\frac{t}{2}}\times\sec^2\frac{t}{2}\times\frac{1}{2}\right]$$

$$= a\left[-\sin t + \frac{1}{\frac{\sin t/2}{\cos t/2}}\times\frac{1}{\cos^2\frac{t}{2}}\times\frac{1}{2}\right]$$

$$= a\left[-\sin t + \frac{1}{2\sin\frac{t}{2}\cdot\cos\frac{t}{2}}\right]$$

$$= a\left[-\sin t + \frac{1}{\sin t}\right] \quad [\because \sin 2\theta = 2\sin\theta\cos\theta]$$

$$= a\left(\frac{1-\sin^2 t}{\sin t}\right)$$

$$\Rightarrow \quad \frac{dx}{dt} = a\left(\frac{\cos^2 t}{\sin t}\right) \quad [\because 1-\sin^2\theta = \cos^2\theta]$$ ....(iii)

Again,on differentiating both sides of (ii) w.r.t t, we get,

$\frac{dy}{dt}$ = a cos t...............(iv)

Therefore, $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{a\cos t}{a\left(\frac{\cos^2 t}{\sin t}\right)}$ [from Eqs(iii) and (iv)]

$= \frac{a\cos t}{a\cos^2 t} \times \sin t$ = tan t

Therefore,on differentiating both sides of above equation w.r.t x, we get,

$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(\tan t)$

$= \frac{d}{dt}(\tan t)\frac{dt}{dx}\left[\because \frac{d}{dx}f(t) = \frac{d}{dt}f(t)\cdot\frac{dt}{dx}\right]$

$\Rightarrow \quad \frac{d^2y}{dx^2} = \sec^2 t \times \frac{\sin t}{a\cos^2 t}$ [From Eq.(iii)]

$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{\sin t \sec^4 t}{a}$

Therefore, on putting t = $\frac{\pi}{3}$,we get,

$\left[\frac{d^2y}{dx^2}\right]_{t=\frac{\pi}{3}} = \frac{\sin\frac{\pi}{3}\times\sec^4\frac{\pi}{3}}{a} = \frac{\frac{\sqrt{3}}{2}\times(2)^4}{a}$

$= \frac{8\sqrt{3}}{a}$

Section D

32. According to the question ,

Given parabola is y^2 = x......(i)

vertex of parabola is (0, 0)

axis of parabola lies along X-axis.

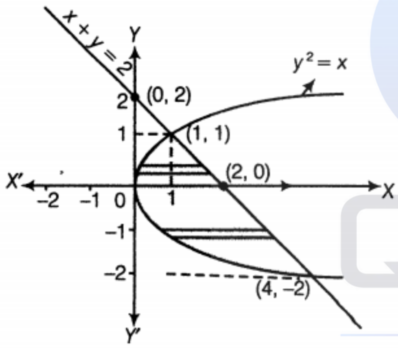
Given equation of line is x + y = 2.......(ii)

For, x + y = 2

x	2	0
y	0	2

So, line passes through the points (2, 0) and (0, 2).

Now, let us sketch the graph of given curve and line as shown below:



On putting x = 2 - y from Eq. (ii) in Eq. (i), we get

$y^2 = 2 - y$

$\Rightarrow y^2 + y - 2 = 0$

$\Rightarrow y^2 + 2y - y - 2 = 0$

$\Rightarrow y(y + 2) - 1(y + 2) = 0$

$\Rightarrow (y - 1)(y + 2) = 0$

$\therefore$ y = 1 or -2

When y = 1, then x = 2 - y = 1

When y = -2, then x = 2 - y = 2 - (- 2) = 4

So, points of intersection are (1, 1) and (4, - 2).

Now, required area = $\int_{-2}^{1}\left[x_{(line)} - x_{\text{(parabola)}}\right]dy$

$= \int_{-2}^{1}\left(2 - y - y^2\right)dy$

$= \left[2y - \frac{y^2}{2} - \frac{y^3}{3}\right]_{-2}^{1}$

$= \left(2 - \frac{1}{2} - \frac{1}{3}\right) - \left(-4 - 2 + \frac{8}{3}\right)$

$= 2 - \frac{5}{6} + 6 - \frac{8}{3}$

$= 8 - \frac{5}{6} - \frac{8}{3}$

$= \frac{48-5-16}{6}$

$= \frac{48-21}{6}$

$= \frac{27}{6}$

$= \frac{9}{2}$ sq units.

33. A = {1, 2, 3, 4, 5} and R = {(a, b) : |a - b| is even}, then R = {(1, 3), (1, 5), (3, 5), (2, 4)}

1. For (a, a), |a - a| = 0 which is even. $\therefore$ R is reflexive.
   If |a - b| is even, then |b - a| is also even. $\therefore$ R is symmetric.
   Now, if |a - b| and |b - c| is even then |a - b + b - c| is even
   $\Rightarrow$ |a - c| is also even. $\therefore$ R is transitive.
   Therefore, R is an equivalence relation.
2. Elements of {1, 3, 5} are related to each other.
   Since |1 - 3| = 2, |3 - 5| = 2, |1 - 5| = 4 all are even numbers
   $\Rightarrow$ Elements of {1, 3, 5} are related to each other.
   Similarly elements of (2, 4) are related to each other.
   Since |2 - 4| = 2 an even number, then no element of the set {1, 3, 5} is related to any element of (2, 4).
   Hence no element of {1, 3, 5} is related to any element of {2, 4}.

OR

A = R – {3} and B = R – {1} and $f(x) = \left(\frac{x-2}{x-3}\right)$

Let $x_1, x_2 \in$ A, then $f(x_1) = \frac{x_1-2}{x_1-3}$ and $f(x_2) = \frac{x_2-2}{x_2-3}$

Now, for $f(x_1) = f(x_2)$

$\Rightarrow \frac{x_1-2}{x_1-3} = \frac{x_2-3}{x_2-3}$

$\Rightarrow (x_1-2)(x_2-3) = (x_2-2)(x_1-3)$

$\Rightarrow x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 2x_1 - 3x_2 + 6$

$\Rightarrow -3x_1 - 2x_2 = -2x_1 - 3x_2$

$= x_1 = x_2$

$\therefore f$ is one-one function.

Now $y = \frac{x-2}{x-3}$

$\Rightarrow y(x-3) = x-2$

$\Rightarrow xy - 3y = x - 2$

$\Rightarrow x(y-1) = 3y-2$

$\Rightarrow x = \frac{3y-2}{y-1}$

$\therefore f\left(\frac{3y-2}{y-1}\right) = \frac{\frac{3y-2}{y-1}-2}{\frac{3y-2}{y-1}-3} = \frac{3y-2-2y+2}{2y-2-3y+3} = y$

$\Rightarrow f(x) = y$

Therefore, f is an onto function.

34. We have, A = $\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, B = $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$

$\therefore$ BA = $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$ = 6I

$\therefore$ B^{-1} = $\frac{A}{6} = \frac{1}{6}$ A = $\frac{1}{6} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$

Also, x - y = 3, 2x + 3y + 4z = 17 and y + 2z = 7

$\Rightarrow \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$

$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$

$= \frac{1}{6} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$ [using Eq. (i)]

$$= \frac{1}{6}\begin{bmatrix} 6+34-28 \\ -12+34-28 \\ 6-17+35 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

$\therefore$ x = 2, y = -1 and z = 4

35. $\vec{a}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{b}_1 = \hat{i} - 3\hat{j} + 2\hat{k}$

$\vec{a}_2 = 4\hat{i} + 5\hat{j} + 6\hat{k}$

$\vec{b}_2 = 2\hat{i} + 3\hat{j} + \hat{k}$

$\vec{a}_2 - \vec{a}_1 = 3\hat{i} + 3\hat{j} + 3\hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix}$$

$= -9\hat{i} + 3\hat{j} + 9\hat{k}$

$(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = (3\hat{i} + 3\hat{j} + 3\hat{k})(-9\hat{i} + 3\hat{j} + 9\hat{k}) = -27 + 9 + 27 = 9$

$$d = \left| \frac{(\vec{a}_2 - \vec{a}_2) \cdot (\vec{b}_1 \times \vec{b}_2)}{\left| \vec{b}_1 \times \vec{b}_2 \right|} \right|$$

$$= \left| \frac{9}{3\sqrt{19}} \right| = \frac{3}{\sqrt{19}}$$

OR

Let P be the point with position vector $\vec{p} = 3\hat{\imath} + \hat{\jmath} + 2\hat{k}$ and M be the image of P in the plane $\vec{r} \cdot (2\hat{\imath} - \hat{\jmath} + \hat{k}) = 4$.

In addition, let Q be the foot of the perpendicular from P on to the given plane. So, Q is the midpoint of PM.

Direction ratios of PM are proportional to 2, –1, 1 as PM is normal to the plane and parallel to $2\hat{\imath} - \hat{\jmath} + \hat{k}$.

Recall the vector equation of the line passing through the point with position vector$\vec{r}$ and parallel to vector $\vec{b}$ is given by

$\vec{r} = \vec{a} + \lambda\vec{b}$

Here,$\vec{a} = 3\hat{\imath} + \hat{\jmath} + 2\hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$

Hence, the equation of PM is

$\vec{r} = (3\hat{i} + \hat{j} + 2\hat{k}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$

$\therefore \vec{r} = (3 + 2\lambda)\hat{i} + (1 - \lambda)\hat{j} + (2 + \lambda)\hat{k}$

Let the position vector of M be $\vec{m}$. As M is a point on this line, for some scalar α, we have

$\Rightarrow \vec{m} = (3 + 2\alpha)\hat{i} + (1 - \alpha)\hat{j} + (2 + \alpha)\hat{k}$

Now, let us find the position vector of Q, the midpoint of PM.

Let this be $\vec{q}$.

Using the midpoint formula, we have

$\vec{q} = \frac{\vec{p} + \vec{m}}{2}$

$\Rightarrow \vec{q} = \frac{[3\hat{i} + \hat{j} + 2\hat{k}] + [(3+2\alpha)\hat{i} + (1-\alpha)\hat{j} + (2+\alpha)\hat{k}]}{2}$

$\Rightarrow \vec{q} = \frac{(3 + (3+2\alpha))\hat{i} + (1 + (1-\alpha))\hat{j} + (2 + (2+\alpha))\hat{k}}{2}$

$\Rightarrow \vec{q} = \frac{(6+2\alpha)\hat{i} + (2-\alpha)\hat{j} + (4+\alpha)\hat{k}}{2}$

$\therefore \vec{q} = (3 + \alpha)\hat{i} + \frac{(2-\alpha)}{2}\hat{j} + \frac{(4+\alpha)}{2}\hat{k}$

This point lies on the given plane, which means this point satisfies the plane equation $\vec{r} \cdot (2\hat{\imath} - \hat{\jmath} + \hat{k}) = 4$.

$\Rightarrow \left[(3 + \alpha)\hat{i} + \frac{(2-\alpha)}{2}\hat{j} + \frac{(4+\alpha)}{2}\hat{k}\right] \cdot (2\hat{\imath} - \hat{j} + \hat{k}) = 4$

$\Rightarrow 2(3 + \alpha) - \left(\frac{2-\alpha}{2}\right)(1) + \left(\frac{4+\alpha}{2}\right)(1) = 4$

$\Rightarrow 6 + 2\alpha + \frac{4+\alpha-(2-\alpha)}{2} = 4$

$\Rightarrow 2\alpha + (1 + \alpha) = -2$

$\Rightarrow 3\alpha = -3$

$\therefore \alpha = -1$

We have the image $\vec{m} = (3 + 2\alpha)\hat{i} + (1 - \alpha)\hat{j} + (2 + \alpha)\hat{k}$

$\Rightarrow \overrightarrow{\mathrm{m}} = [3 + 2(-1)]\hat{\mathrm{i}} + [1 - (-1)]\hat{\mathrm{j}} + [2 + (-1)]\hat{\mathrm{k}}$

$\therefore \overrightarrow{\mathrm{m}} = \hat{\imath} + 2\hat{\jmath} + \hat{\mathrm{k}}$

Therefore, the image is (1, 2, 1)

The foot of the perpendicular $\overrightarrow{\mathrm{q}} = (3 + \alpha)\hat{\mathrm{i}} + \frac{(2-\alpha)}{2}\hat{\mathrm{j}} + \frac{(4+\alpha)}{2}\hat{\mathrm{k}}$

$\Rightarrow \overrightarrow{\mathrm{q}} = [3 + (-1)]\hat{\mathrm{i}} + \left[\frac{2-(-1)}{2}\right]\hat{\mathrm{j}} + \left[\frac{4+(-1)}{2}\right]\hat{\mathrm{k}}$

$\therefore \overrightarrow{\mathrm{q}} = 2\hat{\mathrm{i}} + \frac{3}{2}\hat{\mathrm{j}} + \frac{3}{2}\hat{\mathrm{k}}$

Thus, the position vector of the image is $\hat{i} + 2\hat{j} + \hat{k}$ and that of the foot of the perpendicular is $2\hat{\imath} + \frac{3}{2}\hat{\jmath} + \frac{3}{2}\hat{k}$

Section E

36. i. Let P be the event that the shell fired from A hits the plane and Q be the event that the shell fired from B hits the plane. The following four hypotheses are possible before the trial, with the guns operating independently:

E_1 = PQ, $E_2 = \bar{P}\bar{Q}$, $E_3 = \bar{P}Q$, $E_4 = P\bar{Q}$

Let E = The shell fired from exactly one of them hits the plane.

$P(E_1) = 0.3 \times 0.2 = 0.06$, $P(E_2) = 0.7 \times 0.8 = 0.56$, $P(E_3) = 0.7 \times 0.2 = 0.14$, $P(E_4) = 0.3 \times 0.8 = 0.24$

$P\left(\frac{E}{E_1}\right) = 0\,, P\left(\frac{E}{E_2}\right) = 0\,, P\left(\frac{E}{E_3}\right) = 1\,, P\left(\frac{E}{E_4}\right) = 1$

$P(E) = P(E_1) \cdot P\left(\frac{E}{E_1}\right) + P(E_2) \cdot P\left(\frac{E}{E_2}\right) + P(E_3) \cdot P\left(\frac{E}{E_3}\right) + P(E_4) \cdot P\left(\frac{E}{E_4}\right)$

= 0.14 + 0.24 + = 0.38

ii. By Bayes' Theorem, $P\left(\frac{E_3}{E}\right) = \frac{P(E_3) \cdot P\left(\frac{E}{E_3}\right)}{P(E_1) \cdot P\left(\frac{E}{E_1}\right) + P(E_2) \cdot P\left(\frac{E}{E_2}\right) + P(E_3) \cdot P\left(\frac{E}{E_3}\right) + P(E_4) \cdot P\left(\frac{E}{E_4}\right)}$

$= \frac{0.14}{0.38} = \frac{7}{19}$

NOTE: The four hypotheses form the partition of the sample space and it can be seen that the sum of their probabilities is 1. The hypotheses E_1 and E_2 are actually eliminated as $P\left(\frac{E}{E_1}\right) = P\left(\frac{E}{E_2}\right) = 0$

iii. By Bayes' Theorem, $P\left(\frac{E_4}{E}\right) = \frac{P(E_4) \cdot P\left(\frac{E}{E_4}\right)}{P(E_1) \cdot P\left(\frac{E}{E_1}\right) + P(E_2) \cdot P\left(\frac{E}{E_2}\right) + P(E_3) \cdot P\left(\frac{E}{E_3}\right) + P(E_4) \cdot P\left(\frac{E}{E_4}\right)}$

$= \frac{0.24}{0.38} = \frac{12}{19}$

OR

Let P be the event that the shell fired from A hits the plane and Q be the event that the shell fired from B hits the plane. The following four hypotheses are possible before the trial, with the guns operating independently:

E_1 = PQ, $E_2 = \bar{P}\bar{Q}$, $E_3 = \bar{P}Q$, $E_4 = P\bar{Q}$

Let E = The shell fired from exactly one of them hits the plane.

$P(E_1) = 0.3 \times 0.2 = 0.06$, $P(E_2) = 0.7 \times 0.8 = 0.56$, $P(E_3) = 0.7 \times 0.2 = 0.14$, $P(E_4) = 0.3 \times 0.8 = 0.24$

37. i. Force applied by team A

$= \sqrt{4^2 + 0^2}$

= 4 N

Force applied by team B

$= \sqrt{(-2)^2 + 4^2}$

$= \sqrt{4 + 16}$

$= \sqrt{20}$

$= 2\sqrt{5}$ N

Force applied by team C

$= \sqrt{(-3)^2 + (-3)^2}$

$= \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2}$

Hence, the force applied by team B is maximum.

So, Team 'B' will win.

ii. Sum of force applied by team A, B and C

$= (4 + (-2) + (-3))\hat{i} + (0 + 4 + (-3))\hat{j}$

$= -\hat{i} + \hat{j}$

Magnitude of team combine force

$= \sqrt{(-1)^2 + (1)^2}$

$= \sqrt{2}N$

iii. Force applied by team B

$= \sqrt{(-2)^2 + 4^2}$

$= \sqrt{4 + 16}$

$= \sqrt{20}$

$= 2\sqrt{5}$ N

OR

Force applied by team A

$= \sqrt{4^2 + 0^2}$

= 4 N

38. i. C = $40000h^2 + 5000x^2$

as $x^2h = 250$

$\Rightarrow$ C = $\frac{40000(250)^2}{x^4} + 5000x^2$

ii. $\frac{dC}{dx} = \frac{-160000(250)^2}{x^5} + 10000x$

iii. For minimum cost $\frac{dc}{dx} = 0$

$\Rightarrow 10000x^6 = 250 \times 250 \times 160000$

$\Rightarrow$ x = 10

showing $\frac{d^2C}{dx^2} > 0$ at x = 10

$\therefore$ cost is minimum when x = 10

OR

$\frac{dC}{dx} = \frac{-160000(250)^2}{x^4} + 10000x$

$\frac{dC}{dx} = 0$ gives x = 10

$\frac{dC}{dx} > 0$ in $(10, \infty)$ and $\frac{dC}{dx} < 0$ in (0, 10).

Hence, cost function is neither increasing nor decreasing for x > 0