Section A

1.

(c) 8

Explanation:

8

2.

(c) 8 or -8

Explanation:

8 or -8

Explanation

We know that |Adj A| = $|A|^{n-1}$, n is the order of the matrix.

$\because$ 64 = $|A|^{3-1}$

$|A|^2 = 64$

$|A| = \pm 8$

3.

(b) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm 2A$

Explanation:

$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm 2A$

Explanation:

A = $\frac{1}{2}$ $[x_1 (y_2 - y_3) - x_2 (y_1 - y_3) + x_3 (y_1 - y_2)]$

2A = $[x_1 (y_2 - y_3) - x_2 (y_1 - y_3) + x_3 (y_1 - y_2)]$

4.

(b) 5

Explanation:

5

Explanation

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x+y+z \\ y+z \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$$

$\Rightarrow$ x+y+z =6 ------(i)

y+z = 3 -----------(ii)

z=2 --------------(iii)

from (ii) and (iii)

y + 2 = 3

y = 1

x+ y+ z = 6

x + 3 = 6

x=3

(2x + y - z)

= 2(3)+1-2

= 6+1-2

= 7-2

=5

5.

(b) $\frac{x-2}{1}=\frac{y+1}{2}=\frac{z}{-2}$

Explanation:

$\frac{x-2}{1}=\frac{y+1}{2}=\frac{z}{-2}$

6.

(d) (1 + sin x)(1 + cos y) = C

Explanation:

Given cos x (1+cos y) dx – sin y (1+sin x) dy = 0

Let 1 + cos y = t and 1 + sin x = u

On differentiating both equations, we obtain

-sin y dy = dt and cos x dx = du

Put this in the first equation

t du + u dt = 0

$-\frac{du}{u}=\frac{dt}{t}$

-log u = log t + C

log u + log t = C

log ut = C

ut = C

(1 + sin x)(1 + cos y) = C

7. **(a)** linear function

Explanation:

linear function

8. **(a)** 3

Explanation:

3

9.

(c) $\frac{1}{3}e^{x^3}+C$

Explanation:

Let $I=\int x^2e^{x^3}dx$

Also, let x^3 = t, ⇒ $3x^2$dx = dt

Thus,

$\Rightarrow I=\frac{1}{3}\int e^t dt$

$=\frac{1}{3}(e^t)+C$

$=\frac{1}{3}\left(e^{x^3}\right)+C$

10.

(d) 1, 1

Explanation:

Given, $\begin{bmatrix} x & y \\ 3y & x \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

On multiplying the matrices of left side, we get

$\begin{bmatrix} x \cdot 1 + 2 \cdot y \\ 3y \cdot 1 + 2 \cdot x \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x + 2y \\ 3y + 2x \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

Now, equating the corresponding elements, we get

x + 2y = 3 ...(i)

and 3y + 2x = 5 ...(ii)

On solving Eqs. (i) and (ii), we get

x = 1 and y = 1

11.

(b) 60

Explanation:

Here the objective function is given by : F = 4x +6y .

Corner points	Z = 4x +6 y
(0, 2)	12(Min.)
(3,0)	12.(Min.)
(6,0)	24
(6 , 8)	72
(0 , 5)	30

Maximum of F – Minimum of F = 72 – 12 = 30 .

12.

(c) $\vec{a} \cdot \vec{b} \leq |\vec{a}||\vec{b}|$

Explanation:

The correct answer is $\vec{a} \cdot \vec{b} \leq |\vec{a}||\vec{b}|$. This follows from the Cauchy-Schwarz inequality, which states that the dot product of two vectors is less than or equal to the product of their magnitudes. Equality holds only when the vectors are parallel.

13. **(a)** (2, 4)

Explanation:

(2, 4)

14.

(d) $\frac{2}{3}$

Explanation:

$\frac{2}{3}$

15.

(c) sec x

Explanation:

Given that $\frac{dy}{dx}$ + y tan x - sec x = 0

Here, P = tan x, Q = sec x

IF = $e^{\int P dx} = e^{\int \tan x dx}$

$= e^{\log \sec x}$

$= \sec x$

16. **(a)** [0, 12]

Explanation:

$|\lambda\vec{a}| = |\lambda||\vec{a}|$

$= |\lambda| \times 4$

$= 4|\lambda| \geqslant 0$

When

$\lambda = -3 \Rightarrow 41 - 31 = 12$

$\lambda = 0 \Rightarrow 4 \times 0 = 0.$

$1 \rightarrow \vec{a}$ lie $b/w.$

$|\lambda|\vec{a} \Rightarrow [-0, 12]$

$[0, 12]$

17.

(d) No value

Explanation:

No value

18.

(d) 0, 1, 0

Explanation:

0, 1, 0

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Circumference of circle with radius r is given by C = 2πr

Differentiating w.r.t. 't', we get

$\frac{dC}{dt} = 2\pi\frac{dr}{dt}$

Given, $\frac{dC}{dt} = 10$ cm/s

$\therefore 10 = 2\pi\frac{dr}{dt}$

$\Rightarrow \frac{dr}{dt} = \frac{5}{\pi}$cm/s

Now, Area of circle, A = πr^2

$\therefore \frac{dA}{dt} = 2\pi r\frac{dr}{dt}$

Substituting r = 3 cm and $\frac{dr}{dt} = \frac{5}{\pi}$cm/s, we get

$\frac{dA}{dt} = 2\pi \times 3 \times \frac{5}{\pi}$

$\therefore \frac{dA}{dt} = 30 \text{ cm}^2/\text{s}$

20.

(d) A is false but R is true.

Explanation:

Assertion: Let A = {1, 2, 3, 4, 5, 6}

A relation R is defined on set A is

R = {(a, b) : b = a + 1}

$\therefore$ R = {(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)}

Now, 6 $\in$ A but (6, 6) $\notin$ R

$\therefore$ R is not reflexive.

Reason: Given set A = {1, 2, 3}

A relation R on A is defined as

R = {(1, 2), (2, 1)}

$\because$ (1, 2) $\in$ R and (2, 1) $\in$ R.

So, R is symmetric.

Section B

21. We have, $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)+\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)+\tan^{-1}\left[\sin\left(\frac{-\pi}{2}\right)\right]$.

$=\tan^{-1}\left(\tan\frac{5\pi}{6}\right)+\cot^{-1}\left(\cot\frac{\pi}{3}\right)+\tan^{-1}(-1)$.

$=\tan^{-1}\left[\tan\left(\pi-\frac{\pi}{6}\right)\right]+\cot^{-1}\left[\cot\left(\frac{\pi}{3}\right)\right]+tan^{-1}\left[\tan\left(\pi-\frac{\pi}{4}\right)\right]$

$=\tan^{-1}\left(-\tan\frac{\pi}{6}\right)+\cot^{-1}\left(\cot\frac{\pi}{3}\right)+\tan^{-1}\left(-\tan\frac{\pi}{4}\right)\left[\begin{array}{c}\because\tan^{-1}(\tan x)=x, x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)\\ \cot^{-1}(\cot x)=x, x\in(0,\ \pi)\\ and\ \tan^{-1}(-x)=-\tan^{-1}x\end{array}\right]$

$=-\frac{\pi}{6}+\frac{\pi}{3}-\frac{\pi}{4}=\frac{-2\pi+4\pi-3\pi}{12}$

$=\frac{-5\pi+4\pi}{12}=\frac{-\pi}{12}$

OR

We know that the range of principal value of cosec^{-1} is $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$ - [0]

Let $\text{cosec}^{-1}(-1)=\theta$. Then we have, cosec θ = -1

cosec θ = -1 = -cosec $\frac{\pi}{2}$ = cosec $\left(\frac{-\pi}{2}\right)$

$\therefore\theta=\frac{-\pi}{2}\in\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$ - [0]

Hence, the principal value of $\text{cosec}^{-1}(-1)$ is equal to $\frac{-\pi}{2}$

22. Given, $C(x) = 0.005x^3 - 0.02x^2 + 30x + 50$

On differentiating both sides w.r.t. x, we get

$\frac{dC}{dx}=0.005\left(3x^2\right)-0.02(2x)+30$

$= 0.015x^2 - 0.04x + 30$

On putting x = 300, we get

$\frac{dC}{dx}=0.015(300)^2-0.04(300)+30$

= 1350 - 12 + 30 = 1368

Therefore,the marginal cost of food for 300 students is Rs 1368.

By providing free mid-day meals to the students of primary classes, care and concern is shown towards their health and nutritional status.

23. Given:

$f(x) = x^3 - ax$

$f'(x)=3x^2-a$

Given that f(x) is an increasing function

$\therefore f'(x)>0$ for all $x\in R$

$\Rightarrow 3x^2-a>0$ for all $x\in R$

$\Rightarrow a<3x^2$ for all $x\in R$

But the last value of $3x^2 = 0$ for x = 0

$\therefore a\le 0$

OR

Let A denote the area of rectangle at instant t.

$\therefore$ A = xy (area of rectangle)

$\Rightarrow\left.\begin{array}{l}\frac{dx}{dt}=-5\text{ cm/min}\\ \frac{dy}{dt}=4\text{ cm/min}\end{array}\right\}$ (given),

$\frac{dA}{dt}=x\frac{dy}{dt}+y\frac{dx}{dt}$

$\Rightarrow\left(\frac{dA}{dt}\right)_{x=5,y=B} = 5\times 4 + 8\times(-5)\ cm^2/min$

$\Rightarrow\frac{dA}{dt}=(20-40)\text{cm}^2/\text{min}$

$\Rightarrow\frac{dA}{dt}=-20\text{ cm}^2/\text{min}$

Here, (-) ve sign shows that area is decreasing at the rate of 20 cm^2/min.

24. We have

$I=\int\frac{x^3+x}{x^4-9}dx=\int\frac{x^3}{x^4-9}dx+\frac{xdx}{x^4-9}=I_1+I_2$

Now $I_1 = \int \frac{x^3}{x^4-9}$

Put t = x^4 - 9 so that $4x^3$ dx = dt. Therefore

$I_1 = \frac{1}{4}\int \frac{dt}{t} = \frac{1}{4}\log|t| + C_1 = \frac{1}{4}\log|x^4-9| + C_1$

Again, $I_2 = \int \frac{x\,dx}{x^4-9}$

Put x^2 = u so that 2x dx = du. Then

$I_2 = \frac{1}{2}\int \frac{du}{u^2-(3)^2} = \frac{1}{2\times 6}\log\left|\frac{u-3}{u+3}\right| + C_2$

$= \frac{1}{12}\log\left|\frac{x^2-3}{x^2+3}\right| + C_2$

Thus $I = I_1 + I_2$

$= \frac{1}{4}\log|x^4-9| + \frac{1}{12}\log\left|\frac{x^2-3}{x^2+3}\right| + C$

25. It is given that f(x) = $\cos^2$ x + sin x, $x \in [0,\pi]$

f'(x) = 2 cos x (-sin x) + cos x

= -2 sin x cos x + cos x

Now, if f'(x) = 0

$\Rightarrow$ 2 sin x cos x = cos x

$\Rightarrow$ cos x (2 sin x - 1) = 0

$\Rightarrow$ sin x = $\frac{1}{2}$ or cos x = 0

$\Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$ or $\frac{\pi}{2}$ as x $\in [0,\pi]$

Next, evaluating the value of f at critical points $x = \frac{\pi}{2}$ and $x = \frac{\pi}{6}$ and at the end points of the interval $[0,\pi]$, (i.e. at x = 0 and $x = \pi$), we get,

$f\left(\frac{\pi}{6}\right) = \cos^2\frac{\pi}{6} + \sin\frac{\pi}{6} = \left(\frac{\sqrt{3}}{2}\right)^2 + \frac{1}{2} = \frac{5}{4}$

$f(\frac{5\pi}{6}) = cos^2(\frac{5\pi}{6}) + sin(\frac{5\pi}{6}) = cos^2(\pi-\frac{\pi}{6}) + sin(\pi-\frac{\pi}{6}) = cos^2\frac{\pi}{6} - sin\frac{\pi}{6} = \frac{5}{4}$

f(0) = $\cos^2$ 0 + sin 0 = 1 + 0 = 1

$f(\pi) = \cos^2\pi + \sin\pi$ = $(-1)^2$ + 0 = 1

$f\left(\frac{\pi}{2}\right) = \cos^2\frac{\pi}{2} + \sin\frac{\pi}{2} = 0 + 1 = 1$

Therefore, the absolute maximum value of f is $\frac{5}{4}$ occurring at x = $\frac{\pi}{6}$ and the absolute minimum value of f is 1 occurring at x = 1, $\frac{\pi}{2}$ and π.

Section C

26. Given $\int_0^{\frac{\pi}{4}} \log(1+\tan x)dx$

Let $I = \int_0^{\frac{\pi}{4}} \log(1+\tan x)dx$ .....(i)

as, $\{\int_0^a f(x)dx = \int_0^a f(a-x)dx\}$

$\Rightarrow \mathrm{I} = \int_0^{\frac{\pi}{4}} \log\left[1+\tan\left(\frac{\pi}{4}-\mathrm{x}\right)\right]\mathrm{dx}$

as $\left\{\tan(A-B) = \frac{\tan(A)-\tan(B)}{1+\tan(A)\tan(B)}\right\}$

$\Rightarrow \mathrm{I} = \int_0^{\frac{\pi}{4}} \log\left[1+\frac{\tan\left(\frac{\pi}{4}\right)-\tan(\mathrm{x})}{1+\tan\left(\frac{\pi}{4}\right)\tan(\mathrm{x})}\right]\mathrm{dx}$

$\Rightarrow \mathrm{I} = \int_0^{\frac{\pi}{4}} \log\left[1+\frac{1-\tan(\mathrm{x})}{1+\tan(\mathrm{x})}\right]\mathrm{dx}$

$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log\left[\frac{2}{1+\tan(x)}\right]dx$

$\Rightarrow \mathrm{I} = \int_0^{\frac{\pi}{4}} \log[2]\mathrm{dx} - \int_0^{\frac{\pi}{4}} \log[1+\tan(\mathrm{x})]\mathrm{dx}$

$\Rightarrow \mathrm{I} = \int_0^{\frac{\pi}{4}} \log[2]\mathrm{dx} - \mathrm{I}$ (from (i))

$\Rightarrow 2\mathrm{I} = [\mathrm{x}\log 2]_0^{\frac{\pi}{4}}$

$\Rightarrow 2\mathrm{I} = \frac{\pi}{4}\log 2 - 0$

$\Rightarrow \mathrm{I} = \frac{\pi}{8}\log 2$

27. E_1 : ball drawn from first bag

E_2 : ball drawn from second bag

A : both drawn balls are red

$P(E_1)\ P(E_2) = \frac{1}{2}$

$P(A|E_1) = \frac{5}{8} \times \frac{4}{7} = \frac{20}{56}$

$P(A|E_2) = \frac{2}{8} \times \frac{1}{7} = \frac{2}{56}$

$P(E_1 \mid A) = \frac{\frac{1}{2}\cdot\frac{20}{56}}{\frac{1}{2}\cdot\frac{20}{56}+\frac{1}{2}\cdot\frac{2}{56}}$

$= \frac{\frac{20}{112}}{\frac{22}{112}} = \frac{10}{11}$

28. According to the question, $I = \int \frac{\sin(x-a)}{\sin(x+a)}dx$

Put $x + a = t \Rightarrow$ dx = dt

$\therefore I = \int \frac{\sin(t-a-a)}{\sin t}dt = \int \frac{\sin(t-2a)}{\sin t}dt$

$= \int \frac{\sin t \cos 2a - \cos t \sin 2a}{\sin t}dt$

$[\because sin(A - B) = sinAcosB - cosAsinB]$

$= \int \cos 2a dt - \int \sin 2a \cdot \cot t dt$

$= cos2a[t] - sin2a[log|sint|] + C_1$

$= (x + a)cos2a - sin2alog|sin(x + a)| + C_1$

[put t = x + a)

$= xcos2a - sin2alog|sin(x + a)| + C1$

OR

Given, $I = \int (2x + 5)\sqrt{10 - 4x - 3x^2}dx$

Let, $2x + 5 = A\frac{d}{dx}\left(10 - 4x - 3x^2\right) + B,$

where A and B are constants.

$\Rightarrow 2x + 5 = A(-4 - 6x) + B$ ...(i)

$\Rightarrow 2x + 5 = -6Ax + (B - 4A)$

Comparing the coefficient of x and constant term we get ,

$\Rightarrow -6A = 2 \Rightarrow A = \frac{-1}{3}$

$B - 4A = 5$

$= 5 + 4\left(\frac{-1}{3}\right) = \frac{11}{3}$

$\therefore \quad A = \frac{-1}{3}$ and $B = \frac{11}{3}$

Thus, $(2x + 5) = \frac{-1}{3}(-4 - 6x) + \frac{11}{3}$ [From Eq. (i)]

Now, $I = \frac{-1}{3}\int(-4 - 6x)\sqrt{10 - 4x - 3x^2}dx + \frac{11}{3}\int\sqrt{10 - 4x - 3x^2}dx$

$let\ \ I = \frac{-1}{3}I_1 + \frac{11}{3}I_2$ ...(ii)

Consider $I_1 = \int(-4 - 6x)\sqrt{10 - 4x - 3x^2}dx$

$Put,\ \ 10 - 4x - 3x^2 = t$

$\Rightarrow (-4 - 6x)dx = dt$

$\therefore \quad I_1 = \int\sqrt{t}dt = \frac{2}{3}t^{3/2} + C_1$

$= \frac{2}{3}\left(10 - 4x - 3x^2\right)^{3/2} + C_1$ ...(iii)

consider $I_2 = \int\sqrt{10 - 4x - 3x^2}dx$

$= \sqrt{3}\int\sqrt{-\left(x^2 + \frac{4}{3}x - \frac{10}{3}\right)}dx$

$= \sqrt{3}\sqrt{-\left(x^2 + 2\cdot\frac{2}{3}x + \left(\frac{2}{3}\right)^2 - \left(\frac{2}{3}\right)^2 - \frac{10}{3}\right)}dx$

$= \sqrt{3}\int\sqrt{\frac{34}{9} - \left(x + \frac{2}{3}\right)^2}dx$

$= \sqrt{3}\int\sqrt{\left(\frac{\sqrt{34}}{3}\right)^2 - \left(x + \frac{2}{3}\right)^2}dx$

$= \frac{\sqrt{3}}{2}\left[\left(x + \frac{2}{3}\right)\sqrt{\left(\frac{\sqrt{34}}{3}\right)^2 - \left(x + \frac{2}{3}\right)^2} + \frac{34}{9}\sin^{-1}\left(\frac{\left(x+\frac{2}{3}\right)}{\left(\frac{\sqrt{34}}{3}\right)}\right) + C_2\right]$

$\left[\because \int\sqrt{a^2 - x^2}dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C\right]$

$= \frac{\sqrt{3}}{2}\left[\left(x + \frac{2}{3}\right)\sqrt{\frac{34}{9} - \left(x + \frac{2}{3}\right)^2} + \frac{34}{9}\sin^{-1}\left(\frac{3x+2}{\sqrt{34}}\right) + C_2\right]$...(iv)

From Equations. (ii), (iii) and (iv), we get

$$I=\frac{-2}{9}\left(10-4x-3x^2\right)^{\frac{3}{2}}+\frac{11\sqrt{3}}{6}\left[\left(x+\frac{2}{3}\right)\sqrt{\frac{34}{9}-\left(x+\frac{2}{3}\right)^2}+\frac{34}{9}\sin^{-1}\left(\frac{3x+2}{\sqrt{34}}\right)\right]+C \text{ [where,} C=\frac{-C_1}{3}+\frac{11}{3}C_2 \text{]}$$

29. The given differential equation is,

$\tan x\frac{dy}{dx}$ = 2x tan x + x^2 - y

$\Rightarrow \frac{dy}{dx}+\frac{1}{\tan x}y=\frac{2x\tan x+x^2}{\tan x}$

$\Rightarrow \frac{dy}{dx}$ + (cot x) y = 2x + x^2 cot x

This is a linear differential equation of the form $\frac{dy}{dx}$ + Py = Q

I.F. $=e^{\int Pdx}=e^{\int \cot x\,dx}=e^{\log\sin x}=\sin x$

The solution of the given differential equation is given by

y × (I.F.) = $\int$ Q × (I.F.) dx + C

y × sin x = $\int$ (2x + x^2 + cot x) sin x dx + C

y sin x = $\int$ 2x sin x dx + $\int$ x^2 cos x dx + C

y sin x = $\int$ 2x sin x dx + [x^2 $\int$ cos x dx - $\int$ ($\frac{d}{dx}$$x^2$ × $\int$ cos x dx] + C

y sin x = $\int$ 2x sin x dx + x^2 sin x - $\int$ 2x sin x dx + C

y sin x = x^2 sin x + C

y = x^2 + cosec x × C...(i)

It is given that, y = 0 when x = $\frac{\pi}{2}$

$\therefore$ 0 = $\left(\frac{\pi}{2}\right)^2$ + cosec $\frac{\pi}{2}$ × C

C = $-\frac{\pi^2}{4}$

Putting C = $-\frac{\pi^2}{4}$ in (i), we get

y = $x^2-\frac{\pi^2}{4}$ cosec x

Hence, y = $x^2-\frac{\pi^2}{4}$ cosec x is the required solution.

OR

The given differential equation is,

x$\cos\left(\frac{y}{x}\right)\frac{dy}{dx}$ = y $\cos\left(\frac{y}{x}\right)$ + x

$\Rightarrow \frac{dy}{dx}=\frac{y\cos\left(\frac{y}{x}\right)+x}{x\cos\left(\frac{y}{x}\right)}$

This is a homogeneous differential equation

Put y = vx and $\frac{dy}{dx}=v+x\frac{dv}{dx}$, we get

$v+x\frac{dv}{dx}=\frac{vx\cos v+x}{x\cos v}$

$\Rightarrow v+x\frac{dv}{dx}=\frac{v\cos v+1}{\cos v}$

$\Rightarrow x\frac{dv}{dx}=\frac{v\cos v+1-v\cos v}{\cos v}$

$\Rightarrow x\frac{dv}{dx}=\frac{1}{\cos v}$

$\Rightarrow$ cos v dv = $\frac{1}{x}$ dx

Integrating both sides, we get

$\int$ cos v dv = $\int\frac{1}{x}$ dx

$\Rightarrow$ sin v = log |x| + C

Put v = $\frac{y}{x}$, we get

sin$\frac{y}{x}$ = log |x| + C ...(i)

At x = 1, y = $\frac{\pi}{4}$ ...(given)

Putting C = $\frac{1}{\sqrt{2}}$ in (i), we get

sin$\frac{y}{x}$ = log |x| + $\frac{1}{\sqrt{2}}$

Hence, sin$\frac{y}{x}$ = log x + $\frac{1}{\sqrt{2}}$ is the required solution.

30. We have to maximize Z= 60 x + 15y First, we will convert the given inequations into equations, we obtain the following equations:

x + y = 50, 3x + y = 90, x = 0 and y = 0

Region represented by x + y $\leq$ 50 :

The line x + y = 50 meets the coordinate axes at A(50, 0) and B(0, 50) respectively. By joining these points we obtain the line 3x + 5 y = 15 Clearly (0, 0) satisfies the inequation x + y $\leq$ 50 . Therefore, the region containing the origin represents the solution set of the inequation x + y $\leq$ 50

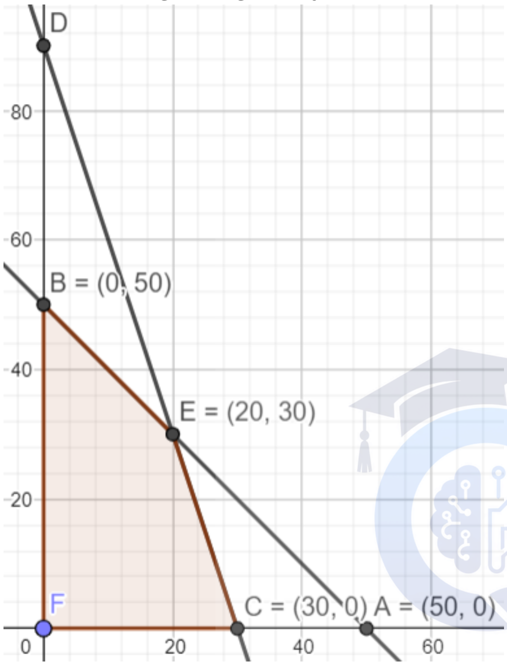
Region represented by 3x + y $\leq$ 90 :

The line 3x + y = 90 meets the coordinate axes at C(30, 0) and D(0, 90) respectively. By joining these points we obtain the line 3x + y = 90 Clearly (0, 0) satisfies the inequation 3x + y $\leq$ 90 . Therefore, the region containing the origin represents the solution set of the inequation 3x + y $\leq$ 90

Region represented by x $\geq$ 0 and y $\geq$ 0 :

since, every point in the first quadrant satisfies these inequations. Therefore, the first quadrant is the region represented by the inequations x $\geq$ 0, and y $\geq$ 0.

The feasible region is given by



The corner points of the feasible region are O(0, 0), C(30, 0) E(20, 30) and B(0, 50)

The values of Z at these corner points are as follows given by

Corner point Z = 60 x + 15 y

O(0, 0) : $60 \times 0 + 15 \times 0 = 0$

C(30, 0) : $60 \times 30 + 15 \times 0 = 1800$

E(20, 30) : $60 \times 20 + 15 \times 30 = 1650$

B(0, 50) : $60 \times 0 + 15 \times 50 = 750$

Therefore, the maximum value of Z is 1800 at the point (30, 0) Hence, x = 30 and y = 0 is the optimal solution of the given LPP. Thus, the optimal value of Z is 1800.This is the required solution.

OR

Converting the given inequations into equations, we obtain the following equations:

x + 2y = 40, 3x + y = 30, 4x + 3y = 60, x = 0 and y = 0

Region represented by x + 2y $\leq$ 40:

The line x + 2y = 40 meets the coordinate axes at A_1 (40, 0) and B_1 (0, 20) respectively. Join these points to obtain the line x + 2y = 40.

Clearly, (0, 0) satisfies the inequation x + 2y $\leq$ 40. So, the region in xy-plane that contains the origin represents the solution set of the given inequation.

Region represented by 3x + y $\geq$ 30:

The line 3x + y = 30 meets x and y axes at A_2 (10, 0) and B_2 (0, 30) respectively. Join these points to obtain this line.

We find that the point O (0, 0) does not satisfy the inequation 3x + y $\geq$ 30.

So, that region in xy-plane which does not contain the origin is the solution set of this inequation.

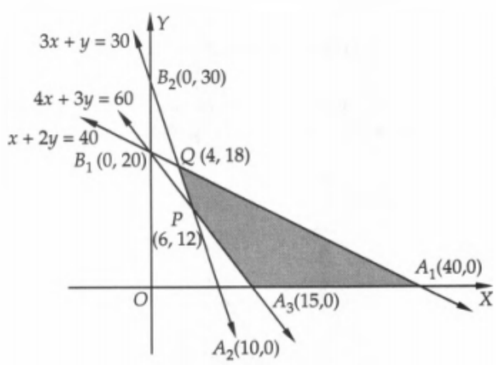
Region represented by 4x + 3y $\geq$ 60:

The line 4x + 3y = 60 meets x and y axes at A_3 (15, 0) and B_1 (0, 20) respectively.

Join these points to obtain the line 4x + 3y = 60. We observe that the point O (0, 0) does not satisfy the inequation 4x + 3y $\geq$ 60. So, the region not containing the origin in xy-plane represents the solution set of the given inequation.

Region represented by x $\geq$ 0, y $\geq$ 0:
Clearly, the region represented by the non-negativity restrictions x $\geq$ 0 and y $\geq$ 0 is the first quadrant in xy-plane.
The shaded region A_3A_1QP in a figure represents the common region of the regions represented by the above inequations.

This region represents the feasible region of the given LPP.



The coordinates of the corner points of the shaded feasible region are A_3 (15, 0), A_1 (40, 0), Q (4,18) and P (6,12). These points have been obtained by solving the equations of the corresponding intersecting lines, simultaneously.
The values of the objective function at these points are given in the following table:

Point (x, y)	Value of the objective function Z = 20x + 10y
A_3 (15, 0)	Z = 20$\times$15 + 10$\times$0 = 300
A_1 (40, 0)	Z = 20$\times$40 + 10$\times$0 = 800
Q(4,18)	Z = 20$\times$4 + 0$\times$18 = 260
P(6,12)	Z = 20$\times$6 + 10$\times$12 = 240

Out of these values of Z, the minimum value is 240 which is attained at point P (6,12). Hence, x = 6, y = 12 is the optimal solution of the given LPP and the optimal value of Z is 240.

31. Let u =$\tan^{-1}\left[\frac{\sqrt{1-x^2}}{x}\right]$

Put x =$\cos\theta \Rightarrow \theta = \cos^{-1}x$

Then, u = $\tan^{-1}\left[\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}\right]$

= $\tan^{-1}\left[\frac{\sqrt{\sin^2\theta}}{\cos\theta}\right]\left[\because \cos^2\theta+\sin^2\theta=1 \Rightarrow \sin^2\theta=1-\cos^2\theta\right]$

= $\tan^{-1}\left[\frac{\sin\theta}{\cos\theta}\right]$

= $\tan^{-1}[\tan\theta]=\theta$

$\Rightarrow \quad u=\cos^{-1}x$

On differentiating both sides w.r.t x, we get

$\frac{du}{dx}=-\frac{1}{\sqrt{1-x^2}}$

Again , let v =$\cos^{-1}\left(2x\sqrt{1-x^2}\right)$

Put x =$\cos\theta \Rightarrow \theta=\cos^{-1}x$

Then, v = $\cos^{-1}\left[2\cos\theta\sqrt{1-\cos^2\theta}\right]$

$=\cos^{-1}[2\cos\theta\sin\theta]\left[\begin{array}{l}\because \sin\theta=\sqrt{1-\cos^2\theta}\\ \Rightarrow \sin^2\theta=1-\cos^2\theta\end{array}\right]$

$=\cos^{-1}[\sin 2\theta]$

$=\cos^{-1}\left[\cos\left(\frac{\pi}{2}-2\theta\right)\right]=\frac{\pi}{2}-2\theta$

$\Rightarrow v=\frac{\pi}{2}-2\cos^{-1}x\ \left[\because \theta=\cos^{-1}x\right]$

On differentiating both sides w.r.t x, we get

$\frac{dv}{dx}=\frac{2}{\sqrt{1-x^2}}$

Now, $\frac{du}{dv}=\frac{du}{dx}\times\frac{dx}{dv}=-\frac{1}{\sqrt{1-x^2}}\times\frac{\sqrt{1-x^2}}{2}$

$=-\frac{1}{2}$

Section D

32. According to the question ,

Given equation of circle is $x^2 + y^2 = 4$......(i)

On differentiating Eq. (i) w.r.t. x , we get

$2x + 2y\frac{dy}{dx} = 0$

$\Rightarrow \quad x + y\frac{d}{dx} = 0$

$\Rightarrow \quad \frac{dy}{dx} = -\frac{x}{y}$

$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{(1,\sqrt{3})} = -\frac{1}{\sqrt{3}}$

$\therefore m = -\frac{1}{\sqrt{3}}$ ('m' is slope of tangent)

Now, equation of tangent at point (1, $\sqrt{3}$) is

$(y - \sqrt{3}) = -\frac{1}{\sqrt{3}}(x - 1)$

$\Rightarrow \sqrt{3}y - 3 = -x + 1$

$\Rightarrow \quad x + \sqrt{3}y = 4$ ....(ii)

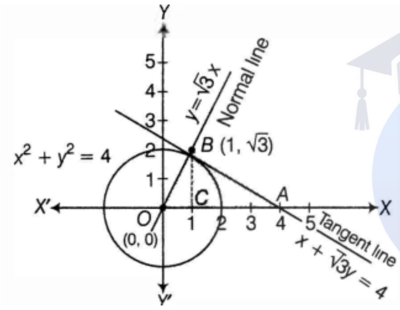
equation of normal passing through point (1, $\sqrt{3}$) and

slope of normal = $\frac{-1}{m_{(tangent)}} = \sqrt{3}$

$(y - \sqrt{3}) = \sqrt{3}(x - 1)$

(y - $\sqrt{3}$) = $\sqrt{3}$x - $\sqrt{3}$ ....(iii)

Now, the Eqs. (i) and (ii) can be represented in the graph as shown below:



On putting y = 0 in Eq. (i), we get

x + 0 = 4

$\Rightarrow$ x = 4

$\therefore$ the tangent line x + $\sqrt{3}$y = 4 cuts the X-axis at A(4,0).

$\therefore$ Required area = Area of shaded region OAB

$= \int_0^1 y_{(\text{ equation of normal})}\, dx + \int_1^4 y_{(\text{equation of tangent})}\, dx$

$= \int_0^1 \sqrt{3}x dx + \int_1^4 \left(\frac{4-x}{\sqrt{3}}\right) dx$

$= \sqrt{3}\left[\frac{x^2}{2}\right]_0^1 + \frac{1}{\sqrt{3}}\left[4x - \frac{x^2}{2}\right]_1^4$

$= \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{3}}\left[16 - \frac{16}{2} - 4 + \frac{1}{2}\right]$

$= \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{3}}\left[12 - \frac{15}{2}\right]$

$= \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{3}}\left[\frac{9}{2}\right]$

$= \frac{\sqrt{3}}{2} + \frac{3\sqrt{3}}{2}$

$= \frac{4\sqrt{3}}{2}$

$= 2\sqrt{3}$sq units.

33. Given that

Let A = {1, 2, 3} and R = {(a, b) : a, b $\in$ A and $\left|a^2 - b^2\right| \leq 5$

Put a = 1, b = 1 $\left|1^2 - 1^2\right| \leq 5, (1, 1)$ is an ordered pair.

Put a = 1, b = 2 $\left|1^2 - 2^2\right| \leq 5, (1, 2)$ is an ordered pair.

Put a = 1, b = 3 $\left|1^2 - 3^2\right| > 5, (1, 3)$ is not an ordered pair.

Put a = 2, b = 1 $\left|2^2 - 1^2\right| \leq 5, (2, 1)$ is an ordered pair.

Put a = 2, b = 2 $\left|2^2 - 2^2\right| \leq 5, (2, 2)$ is an ordered pair.

Put a = 2, b = 3 $\left|2^2 - 3^2\right| \leq 5, (2, 3)$ is an ordered pair.

Put a = 3, b = 1 $|3^2 - 1^2| > 5, (3, 1)$ is not an ordered pair.
Put a = 3, b = 2 $|3^2 - 2^2| \leq 5, (3, 2)$ is an ordered pair.
Put a = 3, b = 3 $|3^2 - 3^2| \leq 5, (3, 3)$ is an ordered pair.
R = {(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 2), (3, 3)}

i. For (a, a) ∈ R
   $|a^2 - a^2| = 0 \leq 5$. Thus, it is reflexive.
ii. Let (a, b) ∈ R
   $(a, b) \in R, |a^2 - b^2| \leq 5$
   $|b^2 - a^2| \leq 5$
   (b, a) ∈ R
   Hence, it is symmetric
iii. Put a = 1, b = 2, c = 3
   $|1^2 - 2^2| \leq 5$
   $|2^2 - 3^2| \leq 5$
   But $|1^2 - 3^2| > 5$
   Thus, it is not transitive

OR

We observe the following properties of f.

Injectivity: Let x, y ∈ R_0 such that f(x) = f(y). Then,

$f(x) = f(y) \Rightarrow \frac{1}{x} = \frac{1}{y} \Rightarrow x = y$

So, f : $R_0 \rightarrow R_0$ is one-one.

Surjectivity: Let y be an arbitrary element of R_0 (co-domain) such that f(x) = y. Then,

$f(x) = y \Rightarrow \frac{1}{x} = y \Rightarrow x = \frac{1}{y}$

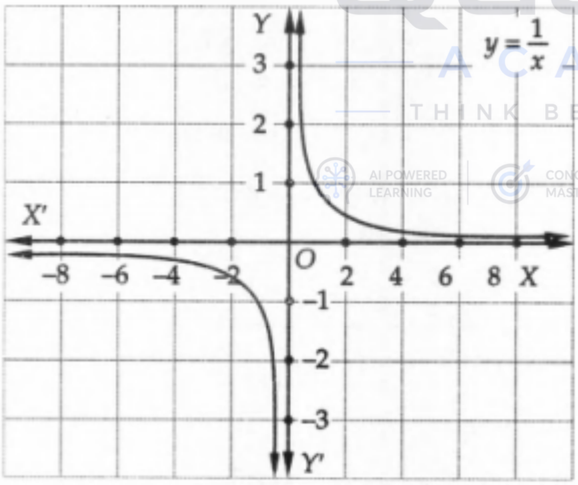
Clearly, $x = \frac{1}{y} \in R_0$ (domain) for all y ∈ R_0 (co-domain).

Thus, for each y ∈ R_0 (co-domain) there exits $x = \frac{1}{y} \in R_0$ (domain) such that $f(x) = \frac{1}{x} = y$

So, f : $R_0 \rightarrow R_0$ is onto.

Hence, f: $R_0 \rightarrow R_0$ is one-one onto.

This is also evident from the graph of f(x) as shown in fig.



Let us now consider f : N → R_0 given by $f(x) = \frac{1}{x}$

For any x, y ∈ N, we find that

f(x) = f(y) ⇒ $\frac{1}{x} = \frac{1}{y} \Rightarrow x = y$

So, f: N → R_0 is one-one.

We find that $\frac{2}{3}, \frac{3}{5}$ etc. in co-domain R_0 do not have their pre-image in domain N. So, f: N →R_0 is not onto.

Thus, f: N →R_0 is one-one but not onto.

34. Let $\frac{1}{x} = u, \frac{1}{y} = v$ and $\frac{1}{z} = w$

2u + 3v + 10w = 4

4u – 6v+ 5w = 1

6u + 9v – 20w = 2

$$A=\begin{bmatrix}2&3&10\\4&-6&5\\6&9&-20\end{bmatrix},X=\begin{bmatrix}u\\v\\w\end{bmatrix}\ B=\begin{bmatrix}4\\1\\2\end{bmatrix}$$

Now, $|A|=\begin{vmatrix}2&3&10\\4&-6&5\\6&9&-20\end{vmatrix}$

= 2[120 - 45] -3[-80 - 30] +10[36 + 36]

= 150 + 330 + 720 = 1200$\neq$ 0

$\Rightarrow$A is non-singular and hence A^{-1} exists.

Now, A_{11} = 75, A_{12} = 110, A_{13} = 72

A_{21} = 150, A_{22} = -100, A_{23} = 0

A_{31} = 75, A_{32} = 30, A_{33} = -2

$$\therefore adjA=\begin{bmatrix}75&150&75\\110&-100&30\\72&0&-24\end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}(adjA)=\frac{1}{1200}\begin{bmatrix}75&150&75\\110&-100&30\\72&0&-24\end{bmatrix}$$

$$X=A^{-1}B$$

$$=\frac{1}{1200}\begin{bmatrix}600\\400\\240\end{bmatrix}$$

$$=\begin{bmatrix}\frac{1}{2}\\\frac{1}{3}\\\frac{1}{5}\end{bmatrix}$$

$$\begin{bmatrix}y\\v\\w\end{bmatrix}=\begin{bmatrix}\frac{1}{2}\\\frac{1}{3}\\\frac{1}{5}\end{bmatrix}$$

$u=\frac{1}{2},v=\frac{1}{3},w=\frac{1}{5}$

$\frac{1}{x}=\frac{1}{2},\frac{1}{y}=\frac{1}{3},\frac{1}{z}=\frac{1}{5}$

x = 2, y = 3, z = 5

35. We have,

$\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}=\lambda$

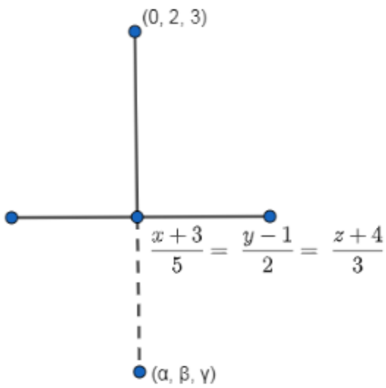
Therefore, the foot of the perpendicular is $(5\lambda-3,2\lambda+1,3\lambda-4)$

The direction ratios of the perpendicular is

$(5\lambda-3-0):(2\lambda+1-2):(3\lambda-4-3)$

$\Rightarrow(5\lambda-3):(2\lambda-1):(3\lambda-7)$

Direction ratio of the line is 5 : 2 : 3



From the direction ratio of the line and direction ratio of its perpendicular, we have

$5(5\lambda-3)+2(2\lambda-1)+3(3\lambda-7)=0$

$\Rightarrow 25\lambda-15+4\lambda-2+9\lambda-21=0$

$\Rightarrow 38\lambda = 38$

$\Rightarrow \lambda = 1$

Therefore, the foot of the perpendicular is (2, 3, -1)

The foot of the perpendicular is the mid-point of the line joining (0, 2, 3) and (a, β, y)

Therefore, we have

$\frac{\alpha+0}{2} = 2 \Rightarrow \alpha = 4$

$\frac{\beta+2}{2} = 3 \Rightarrow \beta = 4$

$\frac{\gamma+3}{2} = -1 \Rightarrow \gamma = -5$

Thus, the imgae is (4, 4, -5)_

OR

Equation of line in vector form

Line I: $\vec{\mathbf{r}} = (\hat{\imath} - \hat{\jmath} + 0\hat{\mathrm{k}}) + \lambda(2\hat{\imath} + 0\hat{\jmath} + \hat{k})$

Line II: $\vec{\mathrm{r}} = (2\hat{\mathrm{f}} - \hat{\mathrm{j}}) + \mu(\hat{\mathrm{i}} + \hat{\mathrm{j}} - \hat{\mathrm{k}})$

Here,

$\overrightarrow{a_1} = \hat{\imath} - \hat{\jmath} + 0\hat{k}$

$\overrightarrow{a_2} = 2\hat{\imath} - \hat{\jmath}$

$\overrightarrow{\mathrm{b}_1} = 2\hat{\mathrm{i}} + 0\hat{\mathrm{j}} + \hat{\mathrm{k}}$

$\overrightarrow{\mathrm{b}_2} = \hat{\mathrm{i}} + \hat{\mathrm{j}} - \hat{\mathrm{k}}$

We know that the shortest distance between lines is

d = $\frac{|(\overrightarrow{\mathrm{a}_2} - \overrightarrow{\mathrm{a}_1})(\overrightarrow{\mathrm{b}_1} \times \overrightarrow{\mathrm{b}_2})|}{|\overrightarrow{\mathrm{b}_1} \times \overrightarrow{\mathrm{b}_2}|}$

$(\overrightarrow{a_2} - \overrightarrow{a_1})$ = $(2\hat{\imath} - \hat{\jmath}) - (\hat{\imath} - \hat{\jmath} + 0\hat{k})$

$(\overrightarrow{a_2} - \overrightarrow{a_1}) = \hat{\imath} + 0\hat{\jmath} + 0\hat{k}$

$\vec{b}_1 \times \vec{b}_2$ = $\begin{vmatrix} \hat{1} & \hat{\jmath} & \hat{k} \\ 2 & 0 & 1 \\ 1 & 1 & -1 \end{vmatrix}$

$\vec{b}_1 \times \vec{b}_2 = (0-1)\hat{\mathrm{i}} - (-2-1)\hat{\mathrm{j}} + (2-0)\hat{\mathrm{k}}$

$\Rightarrow \vec{b}_1 \times \vec{b}_2 = -\hat{1} + 3\hat{\mathrm{j}} + 2\hat{\mathrm{k}}$

$|\overrightarrow{b_1} \times \overrightarrow{b_2}| = \sqrt{(-1)^2 + 3^2 + 2^2}$

$\Rightarrow |\vec{b}_1 \times \vec{b}_2| = \sqrt{14}$

$|(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})|$ = $|(\hat{\imath} + 0\hat{\jmath} + 0\hat{\mathrm{k}})(-\hat{\imath} + 3\hat{\jmath} + 2\hat{\mathrm{k}})|$

$\Rightarrow |(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})| = 1$

Substituting these values in the expression,

d = $\frac{|(\vec{a}_2 - \vec{a}_1)(\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$

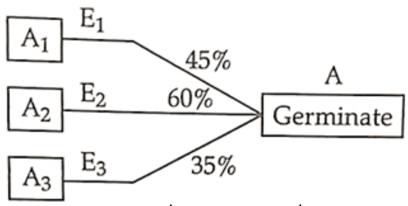
d = $\frac{1}{\sqrt{14}}$

d = $\frac{1}{\sqrt{14}}$ units

Shortest distance d between the lines is not 0. Hence the given lines are not intersecting.

Section E

36. i.



Here, $P(E_1) = \frac{4}{10}$, $P(E_2) = \frac{4}{10}$, $P(E_3) = \frac{2}{10}$

$P\left(\frac{A}{E_1}\right) = \frac{45}{100}, P\left(\frac{A}{E_2}\right) = \frac{60}{100}, P\left(\frac{A}{E_3}\right) = \frac{35}{100}$

$\therefore P(A) = P(E_1) \cdot P\left(\frac{A}{E_1}\right) + P(E_2) \cdot P\left(\frac{A}{E_2}\right) + P(E_3) \cdot P\left(\frac{A}{E_3}\right)$

$= \frac{4}{10} \times \frac{45}{100} + \frac{4}{10} \times \frac{60}{100} + \frac{2}{10} \times \frac{35}{100}$

$= \frac{180}{1000} + \frac{240}{1000} + \frac{70}{100}$
$= \frac{490}{1000}$ = 4.9

ii. Required probability = $P\left(\frac{E_2}{A}\right)$

$= \frac{P(E_2) \cdot P\left(\frac{A}{E_2}\right)}{P(A)}$

$= \frac{\frac{4}{10} \times \frac{60}{100}}{\frac{490}{1000}}$

$= \frac{240}{490} = \frac{24}{49}$

iii. Let,

E_1 = Event for getting an even number on die and

E_2 = Event that a spade card is selected

$\therefore$ P(E_1) = $\frac{3}{6}$

$= \frac{1}{2}$

and P(E_2) = $\frac{13}{52} = \frac{1}{4}$

Then, P($E_1 \cap E_2$) = P(E_1) · P(E_2)

$= \frac{1}{2}, \frac{1}{4} = \frac{1}{8}$

OR

P(A) + P(B) - P(A and B) = P(A)

$\Rightarrow$ P(A) + P(B) - P(A ∩ B) = P(A)

$\Rightarrow$ P(B) - P(A ∩ B) = 0

$\Rightarrow$ P(A ∩ B) = P(B)

$\therefore$ P(A|B) = $\frac{P(A \cap B)}{P(B)}$

$= \frac{P(B)}{P(B)}$

= 1

37. i. Total displacement = $|\vec{d_1}| + |\vec{d_2}| + |\vec{d_3}|$

$|\vec{d_1}| = \sqrt{6^2 + 8^2}$

$= \sqrt{36 + 64}$

$= \sqrt{100}$

= 10 km

$|\vec{d_2}| = \sqrt{3^2 + 4^2}$

$= \sqrt{9 + 16}$

$= \sqrt{25}$

= 5 km

$|\vec{d_3}| = \sqrt{7^2 + 12^2}$

$= \sqrt{49 + 144}$

= 13.89

Total displacement = 10 + 5 + 13.89

= 28.89

$\approx$ 29 km

ii. Speed = $= \frac{\text{Total distance}}{\text{Total time}}$

$= \frac{28.89}{1.5}$

= 19.26 km/hr

iii. Displacement form village to zoo = $d_1 + d_2$

= 10 + 5

= 15 km

OR

Displacement from temple to mall = $d_2 + d_3$

= 5 + 13.89

$= 18.89$

≈ 19 km

38. i. If P is the rent price per apartment and N is the number of rented apartments, the profit is given by $NP - 500\,N = N(P - 500)$

[$\because$ ₹500/month is the maintenance charge for each occupied unit]

ii. Let R be the rent price per apartment and N is the number of rented apartments.

Now, if x be the number of non-rented apartments, then $N(x) = 50 - x$ and $R(x) = 10000 + 250x$

Thus, profit $= P(x) = NR = (50 - x)(10000 + 250x - 500)$

$= (50 - x)(9500 + 250x) = 250(50 - x)(38 + x)$

iii. We have, $P(x) = 250(50 - x)(38 + x)$

Now, $P'(x) = 250[50 - x - (38 + x)] = 250[12 - 2x]$

For maxima/minima, put $P'(x) = 0$

$\Rightarrow 12 - 2x = 0 \Rightarrow x = 6$

Number of apartments are 6.

OR

$P'(x) = 250(12 - 2x)$

$P''(x) = -500 < 0$

$\Rightarrow P(x)$ is maximum at $x = 6$