

Section A

1.

(d) $\frac{\pi}{3}$

Explanation:

Given $A = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$

Therefore, $A' = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$

Also given that $A + A' = I \dots (1)$

(Putting the values in equation (1))

$$\begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} + \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \cos \alpha + \cos \alpha & -\sin \alpha + \sin \alpha \\ \sin \alpha - \sin \alpha & \cos \alpha + \cos \alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 \cos \alpha & 0 \\ 0 & 2 \cos \alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

We know the two matrices are equal only when all their corresponding elements or entries are equal i.e. if $A = B$, then a_{ij} and b_{ij} for all i and j .

This implies,

$$2 \cos \alpha = 1$$

$$\Rightarrow \cos \alpha = \frac{1}{2}$$

$$\Rightarrow \cos \alpha = \cos \frac{\pi}{3} \quad \dots \left(\because \cos \frac{\pi}{3} = \frac{1}{2} \right)$$

$$\Rightarrow \alpha = \frac{\pi}{3}$$

2.

(c) $-1, \frac{11}{5}$

Explanation:

$-1, \frac{11}{5}$

3.

(d) 3

Explanation:

$$y = Ax + A^3$$

Let us find the differential equation representing it so we have to eliminate the constant A

Differentiate with respect to x

$$\Rightarrow \frac{dy}{dx} = A$$

Put back value of A in y

$$\Rightarrow y = \frac{dy}{dx}x + \left(\frac{dy}{dx} \right)^3$$

Now for the degree to exist the differential equation must be a polynomial in some differentials

Here differentials mean $\frac{dy}{dx}$ or $\frac{d^2y}{dx^2}$ or $\frac{d^ny}{dx^n}$

The given differential equation is polynomial in differentials $\frac{dy}{dx}$

The degree of a differential equation is defined as the highest integer power of highest order derivative in the equation

The highest derivative is $\frac{dy}{dx}$ and highest power to it is 3

Hence degree is 3.

4.

(c) $3x^2 e^{x^3}$

Explanation:

Let $y = e^{x^3}$

On differentiating both sides w.r.t. x , we get

$$\begin{aligned}\frac{dy}{dx} &= e^{x^3} \frac{d}{dx}(x^3) \text{ [by chain rule of derivative]} \\ &= e^{x^3} \cdot 3x^2 = 3x^2 e^{x^3}\end{aligned}$$

5.

(c) 3, 1, -2

Explanation:

We have,

$$x - y + z - 5 = 0 = x - 3y - 6$$

$$\Rightarrow x - 3y - 6 = 0$$

$$\&, x - y + z - 5 = 0$$

$$\Rightarrow x = 3y + 6 \dots (i)$$

$$x - y + z - 5 = 0 \dots (ii)$$

From (i) and (ii)

$$\text{We get, } 3y + 6 - y + z - 5 = 0$$

$$\Rightarrow 2y + z + 1 = 0$$

$$\Rightarrow y = \frac{-z-1}{2}$$

$$y = \frac{x-6}{3} \text{ [from (i)]}$$

$$\therefore \frac{x-6}{3} = y = \frac{-z-1}{2}$$

So, the given equation can be re-written as

$$\frac{x-6}{3} = \frac{y}{1} = \frac{z+1}{-2}$$

Hence, the direction ratios of the given line are proportional to 3, 1, -2.

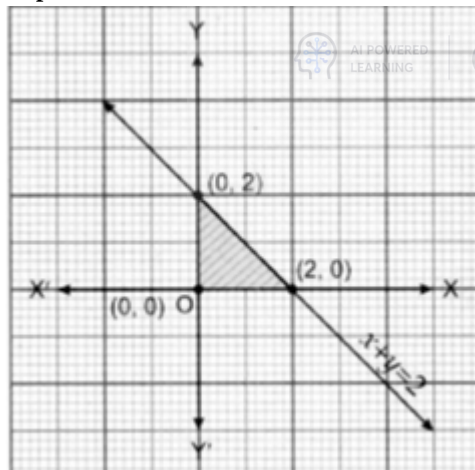
6. (a) 2, degree not defined

Explanation:

2, degree not defined

7. (a) 4

Explanation:



Feasible region is shaded region with corner points (0, 0), (2, 0) and (0, 2).

$$Z(0, 0) = 0$$

$$Z(2, 0) = 4 \leftarrow \text{maximum}$$

$$Z(0, 2) = -2$$

$$Z_{\max} = 4 \text{ and obtained at } (2, 0)$$

8.

(b) [1, 2]

Explanation:

$$\text{Let, } f(x) = \cos^{-1}(2x - 3)$$

$$-1 \leq 2x - 3 \leq 1$$

$$\Rightarrow 2 \leq 2x \leq 4$$

$$\Rightarrow 1 \leq x \leq 2$$

$\therefore x \in [1, 2]$ or domain of x is $[1, 2]$.

9. (a) $\frac{1}{3}(x^2 - 1)^{3/2} + C$

Explanation:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{t\sqrt{t^2-1}} dt = \sec^{-1} t + c$

Therefore ,

$$\Rightarrow \int x \sqrt{x^2 - 1} dx$$

$$\text{Put } x^2 - 1 = t \Rightarrow 2x dx = dt$$

$$\Rightarrow \int \sqrt{t} \frac{dt}{2} = \frac{1}{2} \int t^{\frac{1}{2}} dt$$

$$= \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{(x^2 - 1)^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= \frac{1}{3} (x^2 - 1)^{\frac{3}{2}} + c$$

10.

(b) a skew-symmetric matrix

Explanation:

$$A = \begin{bmatrix} 0 & 5 & -7 \\ -5 & 0 & 11 \\ 7 & -11 & 0 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 0 & -5 & 7 \\ 5 & 0 & -11 \\ -7 & 11 & 0 \end{bmatrix}$$

$$-A = \begin{bmatrix} 0 & -5 & 7 \\ -5 & 0 & 11 \\ 7 & -11 & 0 \end{bmatrix}$$

$$\therefore A^T = -A$$

Then, the given matrix is a skew-symmetric matrix.



11.

(b) $q = 3p$

Explanation:

The maximum value of Z is unique.

It is given that the maximum value of Z occurs at two points $(3, 4)$ and $(0, 5)$

$\therefore$ Value of Z at $(3, 4) =$ Value of Z at $(0, 5)$

$$\Rightarrow p(3) + q(4) = p(0) + q(5)$$

$$\Rightarrow 3p + 4q = 5q$$

$$\Rightarrow q = 3p$$

12.

(b) Infinite

Explanation:

Infinite

13. (a) $\theta = \cos^{-1}\left(\frac{19}{21}\right)$

Explanation:

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If θ is the acute angle between $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \lambda \vec{b}_2$, then cosine of the angle between these two lines is given by

$$\cos \theta = \left| \frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1| |\vec{b}_2|} \right|$$

Here, $\vec{b}_1 = 3\hat{i} + 2\hat{j} + 6\hat{k}$, $\vec{b}_2 = \hat{i} + 2\hat{j} + 2\hat{k}$

Then,

$$\cos \theta = \left| \frac{(3\hat{i} + 2\hat{j} + 6\hat{k}) \cdot (\hat{i} + 2\hat{j} + 2\hat{k})}{|(3\hat{i} + 2\hat{j} + 6\hat{k})| |(\hat{i} + 2\hat{j} + 2\hat{k})|} \right|$$

$$\cos \theta = \left| \frac{19}{\sqrt{49}\sqrt{9}} \right| = \left| \frac{19}{21} \right| \Rightarrow \theta = \cos^{-1} \left(\left| \frac{19}{21} \right| \right)$$

14.

(c) $\frac{3}{8}$

Explanation:

Let A be the event that a person has brown hair, B be the event that a person has brown eyes. Then,

$$P(A) = \frac{40}{100}, P(B) = \frac{25}{100}, P(A \cap B) = \frac{15}{100}$$

$$\text{Required probability} = P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{15}{100}}{\frac{40}{100}} = \frac{3}{8}$$

15.

(b) $e^x + e^{-y} = C$

Explanation:

We have, $\frac{dy}{dx} = e^{x+y}$

$$\Rightarrow \frac{dy}{dx} = e^x \times e^y$$

separating variables

$$\Rightarrow e^{-y} dy = e^x dx$$

Integrating both sides

$$\Rightarrow \int e^{-y} dy = \int e^x dx$$

$$\Rightarrow -e^{-y} = e^x + c$$

$$\Rightarrow e^x + e^{-y} = -c$$

Or,

$$e^x + e^{-y} = c \text{ (c is a constant)}$$

16. (a) $(-\infty, -1)$

Explanation:

$$f(x) = \sin x - ax + b$$

$$\Rightarrow f'(x) = \cos x - a$$

For increasing function

$$f'(x) \geq 0$$

$$\cos x - a \geq 0 \Rightarrow \cos x \geq a$$

$$\text{i.e. } a \leq \cos x \leq \min(\cos x) = -1$$

$$\therefore a \in (-\infty, -1)$$

17.

(c) $-\frac{1}{2a t^3}$

Explanation:

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2a}{2at} = \frac{1}{t}$$

$$\Rightarrow \frac{d^2 y}{dx^2} = \frac{d}{dx} \left(\frac{1}{t} \right) = -\frac{1}{t^2} \frac{dt}{dx} = -\frac{1}{t^2} \cdot \frac{1}{2at} = -\frac{1}{2at^3}$$

18.

(b) $\frac{x-2}{1} = \frac{y+1}{2} = \frac{z}{-2}$



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Explanation:

$$\frac{x-2}{1} = \frac{y+1}{2} = \frac{z}{-2}$$

19. (a) Both A and R are true and R is the correct explanation of A.

Explanation:

Both A and R are true and R is the correct explanation of A.

20. (a) Both A and R are true and R is the correct explanation of A.

Explanation:

$$\text{Let } I = \int \frac{dx}{16+9x^2}$$

$$\text{Let } 3x = t \Rightarrow 3dx = dt$$

$$\Rightarrow dx = \frac{dt}{3}$$

$$\therefore I = \frac{1}{3} \int \frac{dt}{(4)^2+t^2}$$

$$= \frac{1}{3} \times \frac{1}{4} \tan^{-1}\left(\frac{t}{4}\right) + C$$

$$= \frac{1}{12} \tan^{-1}\left(\frac{3x}{4}\right) + C$$

Section B

21. $|A| = p^2 - 4$

$$|A|^3 = 125 \Rightarrow |A| = \sqrt[3]{125} \Rightarrow |A| = 5$$

$$\therefore p^2 - 4 = 5 \Rightarrow p = \pm 3$$

OR

Here, (5, 5), (-5, 1) and (10, 7) are three points ...(Given)

Now, we know that,

Vertices of a triangle are (x_1, y_1) , (x_2, y_2) and (x_3, y_3) , then the area of the triangle is given by:

$$\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Now,

Substituting given value in above formula

$$\Delta = \frac{1}{2} \begin{vmatrix} 5 & 5 & 1 \\ -5 & 1 & 1 \\ 10 & 7 & 1 \end{vmatrix} = 0$$

R.H.S

$$\frac{1}{2} \begin{vmatrix} 5 & 5 & 1 \\ -5 & 1 & 1 \\ 10 & 7 & 1 \end{vmatrix}$$

Expanding along R_1

$$= \frac{1}{2} \left[5 \begin{vmatrix} 1 & 1 \\ 7 & 1 \end{vmatrix} - 5 \begin{vmatrix} -5 & 1 \\ 10 & 1 \end{vmatrix} + 1 \begin{vmatrix} -5 & 1 \\ 10 & 7 \end{vmatrix} \right]$$

$$= \frac{1}{2} [5(-6) - 5(-15) + 1(-45)]$$

$$= \frac{1}{2} [-35 + 75 - 45]$$

$$= 0$$

$$= \text{LHS}$$

Since, Area of triangle is zero

Hence, points are collinear

22. USING PARTIAL FRACTION

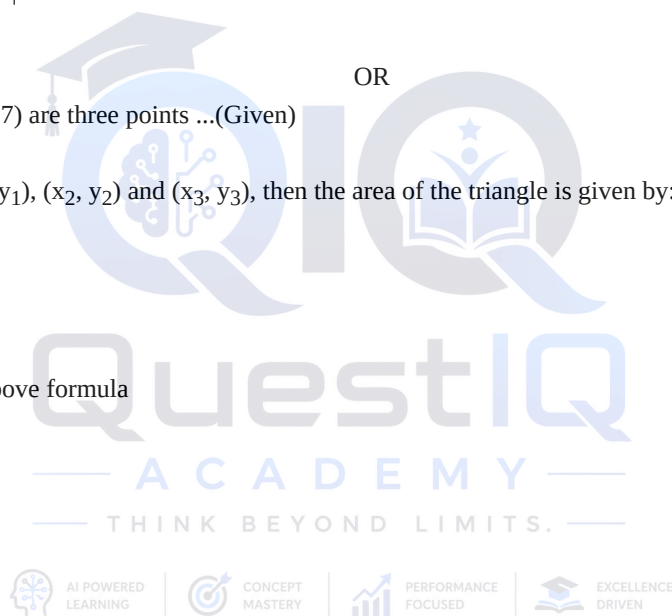
$$\frac{2-x}{(1-x)^2} = \frac{A}{(1-x)} + \frac{B}{(1-x)^2}$$

$$\Rightarrow 2-x = A(1-x) + B$$

For $x = 1$, equation: $1 = B$ i.e. $B = 1$

For $x = 2$, equation: $0 = -A + 1$ i.e. $A = 1$

$$\therefore \frac{2-x}{(1-x)^2}$$



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$$= \frac{1}{(1-x)} + \frac{1}{(1-x)^2}$$

The given equation becomes

$$\int e^x \left[\frac{1}{(1-x)} + \frac{1}{(1-x)^2} \right] dx$$

$$= \int e^x \times \frac{1}{(1-x)^2} dx + \int e^x \times \frac{1}{1-x} dx$$

Taking $f_1(x) = 1/(1-x)$ and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$$\int \frac{e^x}{(1-x)^2} dx + \int \frac{e^x}{1-x} dx$$

$$= \int \frac{e^x}{(1-x)^2} dx + \frac{1}{1-x} \int e^x dx - \int \left[\frac{d}{dx} \left(\frac{1}{1-x} \right) \int e^x dx \right] dx$$

$$= \int \frac{e^x}{(1-x)^2} dx + \frac{e^x}{1-x} - \int \frac{e^x}{(1-x)^2} dx + c$$

$$= \frac{e^x}{1-x} + c, \text{ where } c \text{ is the integrating constant}$$

23. We have

$$f(x) = 2x^3 - 15x^2 + 36x + 1$$

$$\text{or } f'(x) = 6x^2 - 30x + 36 = 6(x-3)(x-2)$$

Note that $f'(x) = 0$ gives $x = 2$ and $x = 3$.

We shall now evaluate the value of f at these points and at the end points of the interval $[1, 5]$, i.e., at $x = 1$, $x = 2$, $x = 3$ and at $x = 5$. So

$$f(1) = 2(1^3) - 15(1^2) + 36(1) + 1 = 24$$

$$f(2) = 2(2^3) - 15(2^2) + 36(2) + 1 = 29$$

$$f(3) = 2(3^3) - 15(3^2) + 36(3) + 1 = 28$$

$$f(5) = 2(5^3) - 15(5^2) + 36(5) + 1 = 56$$

Thus, we conclude that absolute maximum value of f on $[1, 5]$ is 56, occurring at $x = 5$, and absolute minimum value of f on $[1, 5]$ is 24 which occurs at $x = 1$.

OR

$$\text{Given function, } f(x) = \tan x - 4x \Rightarrow f'(x) = \sec^2 x - 4$$

$$\text{When } -\frac{\pi}{3} < x < \frac{\pi}{3}, 1 < \sec^2 x < 2$$

$$\text{Therefore, } 1 < \sec^2 x < 2 \Rightarrow -3 < (\sec^2 x - 4) < 0$$

$$\text{Thus for } -\frac{\pi}{3} < x < \frac{\pi}{3}, f'(x) < 0$$

$$\text{Hence, } f \text{ is strictly decreasing on } \left(-\frac{\pi}{3}, \frac{\pi}{3}\right).$$

24. By Addition theorem of probability, we have

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = P(A) + P(B) - P(A) \times P(B) \quad [\because A \text{ and } B \text{ are independent events}]$$

$$\Rightarrow 0.6 = 0.2 + P(B) - 0.2 \times P(B)$$

$$\Rightarrow 0.6 - 0.2 = P(B)(1 - 0.2)$$

$$\Rightarrow P(B) = \frac{0.6-0.2}{1-0.2}$$

$$\Rightarrow P(B) = \frac{0.4}{0.8}$$

$$\Rightarrow P(B) = \frac{1}{2}$$

$$\Rightarrow P(B) = 0.5$$

$$25. \frac{d(|x|)}{dx} = \frac{d(\sqrt{x^2})}{dx}, x \neq 0$$

$$= \frac{1}{2} (x^2)^{-\frac{1}{2}} \times \frac{d(x^2)}{dx}$$

$$= \frac{1}{2\sqrt{x^2}} 2x = \frac{x}{|x|}$$

Section C

$$26. \text{ Let } I = \int_0^{\pi/2} \frac{1}{2 \cos x + 4 \sin x} dx. \text{ Then,}$$

$$I = \int_0^{\pi/2} \frac{1}{2(1 - \tan^2 \frac{x}{2})} + \frac{4(2 \tan \frac{x}{2})}{1 + \tan^2 \frac{x}{2}} dx$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{1 + \tan^2 \frac{x}{2}}{2 - 2 \tan^2 \frac{x}{2} + 8 \tan \frac{x}{2}} dx = \int_0^{\pi/2} \frac{\sec^2 \frac{x}{2}}{2 - 2 \tan^2 \frac{x}{2} + 8 \tan \frac{x}{2}} dx$$

Using substitution

$$\text{Let } \tan \frac{x}{2} = t. \text{ Then, } d\left(\tan \frac{x}{2}\right) = dt \Rightarrow \frac{1}{2} \sec^2 \frac{x}{2} dx = dt \Rightarrow dx = 2 \frac{dt}{\sec^2 \frac{x}{2}}$$

$$\text{Also, } x = 0 \Rightarrow t = \tan 0 = 0, \text{ and } x = \frac{\pi}{2} \Rightarrow t = \tan \frac{\pi}{4} = 1$$

$$\begin{aligned} \therefore I &= \int_0^1 \frac{\sec^2 \frac{x}{2}}{2-2t^2+8t} \times \frac{2dt}{\sec^2 \frac{x}{2}} = \int_0^1 \frac{1}{1-t^2+4t} dt = \int_0^1 \frac{1}{-(t^2-4t-1)} dt \\ &\Rightarrow I = \int_0^1 \frac{1}{-(t^2-4t+4-4-1)} = \int_0^1 \frac{1}{-\{(t-2)^2-5\}} dt = \int_0^1 \frac{1}{(\sqrt{5})^2-(t-2)^2} dt \\ &\Rightarrow I = \frac{1}{2\sqrt{5}} \left[\log \left| \frac{\sqrt{5}+t-2}{\sqrt{5}-t+2} \right| \right]_0^1 = \frac{1}{2\sqrt{5}} \left[\log \left(\frac{\sqrt{5}-1}{\sqrt{5}+1} \right) - \log \left(\frac{\sqrt{5}-2}{\sqrt{5}+2} \right) \right] \\ &\Rightarrow I = \frac{1}{2\sqrt{5}} \left[\log \left\{ \frac{(\sqrt{5}-1)(\sqrt{5}+2)}{(\sqrt{5}+1)(\sqrt{5}-2)} \right\} \right] = \frac{1}{2\sqrt{5}} \log \left(\frac{3+\sqrt{5}}{3-\sqrt{5}} \right) \\ &\Rightarrow I = \frac{1}{2\sqrt{5}} \log \left(\frac{3+\sqrt{5}}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}} \right) = \frac{1}{2\sqrt{5}} \log \left(\frac{3+\sqrt{5}}{2} \right)^2 = \frac{2}{2\sqrt{5}} \log \left(\frac{3+\sqrt{5}}{2} \right) \\ &\Rightarrow I = \frac{1}{\sqrt{5}} \log \left(\frac{3+\sqrt{5}}{2} \right) \end{aligned}$$

Hence the result

$$\begin{aligned} 27. y &= \cot^{-1} \left[\frac{\sqrt{\left(\cos \frac{x}{2} + \sin \frac{x}{2}\right)^2} + \sqrt{\left(\cos \frac{x}{2} - \sin \frac{x}{2}\right)^2}}{\sqrt{\left(\cos \frac{x}{2} + \sin \frac{x}{2}\right)^2} - \sqrt{\left(\cos \frac{x}{2} - \sin \frac{x}{2}\right)^2}} \right] \left[\because \sqrt{1 \pm \sin x} = \sqrt{\left(\cos \frac{x}{2} \pm \sin \frac{x}{2}\right)^2} \right] \\ &= \cot^{-1} \frac{\cos \frac{x}{2} + \sin \frac{x}{2} + \cos \frac{x}{2} - \sin \frac{x}{2}}{\cos \frac{x}{2} + \sin \frac{x}{2} - \cos \frac{x}{2} + \sin \frac{x}{2}} \\ &= \cot^{-1} \left(\frac{2 \cos \frac{x}{2}}{2 \sin \frac{x}{2}} \right) \\ &= \cot^{-1} \left(\cot \frac{x}{2} \right) \\ &= \frac{x}{2} \\ y &= \frac{x}{2} \\ \frac{dy}{dx} &= \frac{1}{2} \end{aligned}$$

$$28. \text{ According to the question, } I = \int \frac{dx}{x(x^5+3)}$$

Multiplying the numerator and denominator by x^4 , we get

$$= \int \frac{x^4}{x^5(x^5+3)} dx$$

$$\text{Put } t = x^5$$

$$\Rightarrow dt = 5x^4 dx$$

$$= \int \frac{dt}{5t(t+3)}$$

$$\therefore I = \frac{1}{5} \int \frac{dt}{t(t+3)} \dots (i)$$

By using partial fractions,

$$\text{let } \frac{1}{t(t+3)} = \frac{A}{t} + \frac{B}{t+3} \dots (ii)$$

$$1 = A(t+3) + B(t)$$

$$1 = t(A+B) + 3A$$

By comparing the coefficient of t and constant terms, we get

$$3A = 1 \Rightarrow A = \frac{1}{3}$$

$$A + B = 0$$

$$\frac{1}{3} + B = 0$$

$$B = -\frac{1}{3}$$

from (i) and (ii), we get

$$= \frac{1}{5} \int \frac{1}{3} \left[\frac{1}{t} - \frac{1}{t+3} \right] dt$$

$$= \frac{1}{15} [\log |t| - \log |t+3|] + C$$

$$= \frac{1}{15} \log \left| \frac{t}{t+3} \right| + C \left[\because \log m - \log n = \log \frac{m}{n} \right]$$

$$= \frac{1}{15} \log \left| \frac{x^5}{x^5+3} \right| + C \left[\because t = x^5 \right]$$

OR

$$\text{Let } I = \int \frac{1}{\sin x(2+3 \cos x)} dx$$



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multiply and divide numerator by $\sin x$.

$$= \int \frac{\sin x}{\sin^2 x (2+3 \cos x)} dx$$

$$= \int \frac{\sin x}{(1-\cos^2 x)(2+3 \cos x)} dx$$

$$= \int \frac{\sin x}{(1-\cos x)(1+\cos x)(2+3 \cos x)} dx$$

Putting $\cos x = t$

$$\Rightarrow \cos x = t$$

$$\Rightarrow -\sin x dx = dt$$

$$\therefore I = \int \frac{-1}{(1-t)(1+t)(2+3t)} dt$$

$$= \int \frac{1}{(t-1)(t+1)(3t+2)} dt$$

$$\text{Let } \frac{1}{(t-1)(t+1)(3t+2)} = \frac{A}{t-1} + \frac{B}{t+1} + \frac{C}{3t+2}$$

$$\Rightarrow \frac{1}{(t-1)(t+1)(3t+2)} = \frac{A(t+1)(3t+2)+B(t-1)(3t+2)+C(t+1)(t-1)}{(t-1)(t+1)(3t+2)}$$

$$\Rightarrow 1 = (t+1)(3t+2) + B(t-1)(3t+2) + C(t+1)(t-1)$$

Putting $t+1=0$ or $t=-1$

$$\Rightarrow 1 = A \times 0 + B(-1-1)(3 \times -1+2) + C \times 0$$

$$\therefore B = \frac{1}{2}$$

Now, putting $t-1=0$ or $t=1$

$$\Rightarrow 1 = A(1+1)(3+2) + B \times 0 + C \times 0$$

$$\therefore A = \frac{1}{10}$$

Now, putting $3t+2=0$ or $t = -\frac{2}{3}$

$$\Rightarrow 1 = A \times 0 + B \times 0 + C \left(-\frac{2}{3}+1\right) \left(-\frac{2}{3}-1\right)$$

$$\Rightarrow 1 = C \left(\frac{1}{3}\right) \left(-\frac{5}{3}\right)$$

$$\therefore C = \frac{-9}{5}$$

$$\therefore I = \int \frac{1}{10(t-1)} dt + \frac{1}{2} \int \frac{1}{t+1} dt - \frac{9}{5} \int \frac{1}{3t+2} dt$$

$$= \frac{1}{10} \ln |t-1| + \frac{1}{2} \ln |t+1| - \frac{9}{5} \ln \left| \frac{3t+2}{3} \right| + C$$

$$= \frac{1}{10} \ln |t-1| + \frac{1}{2} \ln |t+1| - \frac{3}{5} \ln |3t+2| + C$$

$$= \frac{1}{10} + \ln |\cos x - 1| + \frac{1}{2} \ln |\cos x + 1| - \frac{3}{5} \ln |3 \cos x + 2| + C \quad [\because t = \cos x]$$

29. It is given that $y e^{\frac{x}{y}} dx = \left(x e^{\frac{x}{y}} + y^2 \right) dy$

$$\Rightarrow y e^{\frac{x}{y}} \frac{dx}{dy} = x e^{\frac{x}{y}} + y^2$$

$$\Rightarrow e^{\frac{x}{y}} \left[y \cdot \frac{dx}{dy} - x \right] = y^2$$

$$\Rightarrow e^{\frac{x}{y}} \cdot \frac{\left[y \cdot \frac{dx}{dy} - x \right]}{y^2} = 1 \dots\dots(i)$$

$$\text{Let } e^{\frac{x}{y}} = z$$

Differentiating it w.r.t. y , we get,

$$\frac{d}{dy} \left(e^{\frac{x}{y}} \right) = \frac{dz}{dy}$$

$$\Rightarrow e^{\frac{x}{y}} \cdot \frac{d}{dy} \left(\frac{x}{y} \right) = \frac{dz}{dy}$$

$$\Rightarrow e^{\frac{x}{y}} \cdot \frac{\left[y \cdot \frac{dx}{dy} - x \right]}{y^2} = \frac{dz}{dy} \dots\dots(ii)$$

From equation (i) and equation (ii), we get,

$$\frac{dz}{dy} = 1$$

$$\Rightarrow dz = dy$$

On integrating both sides, we get,

$$z = y + C$$

$$\Rightarrow e^{\frac{x}{y}} = y + C$$

OR

The given differential equation is,

$$(x^2 - 1) \frac{dy}{dx} + 2(x+2)y = 2(x+1)$$

$$\frac{dy}{dx} + \frac{2(x+2)}{(x^2-1)}y = \frac{2(x+1)}{(x^2-1)}$$

It is a linear differential equation. Comparing it with,

$$\frac{dy}{dx} + Py = Q$$

$$P = \frac{2(x+2)}{x^2-1}, Q = \frac{2(x+1)}{(x^2-1)}$$

$$\text{I.F.} = e^{\int P dx}$$

$$= e^{\int \frac{2(x+2)}{(x^2-1)} dx}$$

$$= e^{\int \frac{2x}{x^2-1} dx + 4 \int \frac{1}{x^2-1} dx}$$

$$= e^{\log|x^2-1| + 4x \cdot \frac{1}{2} \log \left| \frac{x-1}{x+1} \right|}$$

$$= e^{\log|x^2-1| + \log \left| \frac{x-1}{x+1} \right|^2}$$

$$= e^{\log \frac{(x^2-1)(x-1)^2}{(x+1)^2}}$$

$$\text{I.F.} = \frac{(x+1)(x-1)(x-1)^2}{(x+1)^2}$$

$$= \frac{(x-1)^3}{(x+1)}$$

Solution of the equation is given by,

$$4 \times (\text{I.F.}) = \int Q \times (\text{I.F.}) dx + c$$

$$\frac{y(x-1)^3}{(x+1)} = 2 \frac{x^2}{2} - 6x + 8 \log|x+1| + c$$

$$\frac{y(x-1)^3}{x+1} = x^2 - 6x + 8 \log|x+1| + c$$

$$y = \frac{x+1}{(x-1)^3} [x^2 - 6x + 8 \log|x+1| + c]$$

30. Given that,

$$\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j}) \text{ and } \vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$$

Comparing the given equations with the equations

$$\vec{r} = \vec{a}_1 + \lambda \vec{b}_1 \text{ and } \vec{r} = \vec{a}_2 + \mu \vec{b}_2$$

We get,

$$\vec{a}_1 = \hat{i} + \hat{j} - \hat{k}$$

$$\vec{a}_2 = 4\hat{i} - \hat{k}$$

$$\vec{b}_1 = 3\hat{i} - \hat{j}$$

$$\vec{b}_2 = 2\hat{i} + 3\hat{k}$$

$$\therefore \vec{a}_2 - \vec{a}_1 = 3\hat{i} - \hat{j}$$

$$\text{and } \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 0 \\ 2 & 0 & 3 \end{vmatrix}$$

$$= -3\hat{i} - 9\hat{j} + 2\hat{k}$$

$$(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = (3\hat{i} - \hat{j}) \cdot (-3\hat{i} - 9\hat{j} + 2\hat{k})$$

$$= -9 + 9$$

$$= 0$$

We observe

$$(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = 0$$

Thus, the given lines intersect.

OR

According to the question ,

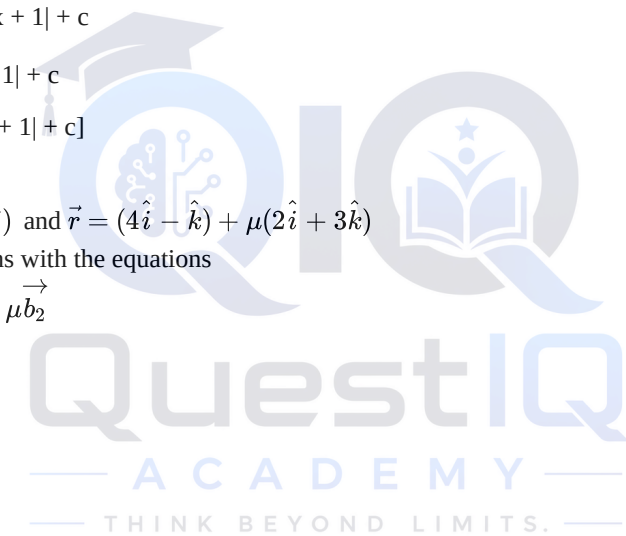
$$\vec{a} \perp (\vec{b} + \vec{c}), \vec{b} \perp (\vec{c} + \vec{a}), \vec{c} \perp (\vec{a} + \vec{b})$$

$$\text{and } |\vec{a}| = 3, |\vec{b}| = 4, |\vec{c}| = 5$$

$$\text{To prove } |\vec{a} + \vec{b} + \vec{c}| = 5\sqrt{2}$$

$$\text{Consider, } |\vec{a} + \vec{b} + \vec{c}|^2 = (\vec{a} + \vec{b} + \vec{c}) \cdot (\vec{a} + \vec{b} + \vec{c}) \left[\because |\vec{x}|^2 = \vec{x} \cdot \vec{x} \right]$$

$$= \vec{a} \cdot \vec{a} + \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} + \vec{b} \cdot \vec{a} + \vec{b} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} + \vec{c} \cdot \vec{b} + \vec{c} \cdot \vec{c}$$



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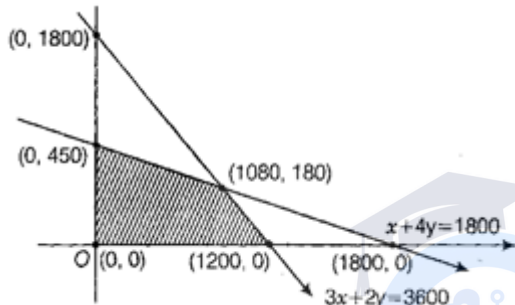
$$\begin{aligned}
&= |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + \vec{a} \cdot (\vec{b} + \vec{c}) + \vec{b} \cdot (\vec{a} + \vec{c}) + \vec{c} \cdot (\vec{a} + \vec{c}) \\
&= |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 0 + 0 + 0 \\
&[\because \vec{a} \perp (\vec{b} + \vec{c})] \\
&\therefore \vec{a} \cdot (\vec{b} + \vec{c}) = 0 \\
&\text{Similarly, } \vec{b} \cdot (\vec{a} + \vec{c}) = 0 \\
&\text{and } \vec{c} \cdot (\vec{a} + \vec{b}) = 0 \\
&= 3^2 + 4^2 + 5^2 = 9 + 16 + 25 \\
&\Rightarrow |\vec{a} + \vec{b} + \vec{c}|^2 = 50 \\
&\Rightarrow |\vec{a} + \vec{b} + \vec{c}| = 5\sqrt{2}
\end{aligned}$$

31. Maximise $Z = 100x + 170y$ subject to

$$3x + 2y \leq 3600, x + 4y \leq 1800, x \geq 0, y \geq 0$$

From the shaded feasible region it is clear that the coordinates of corner points are (0,0), (1200,0), (1080,180) and (0,450).

On solving $x + 4y = 1800$ and $3x + 2y = 3600$ we get $x = 1080$ and $y = 180$.

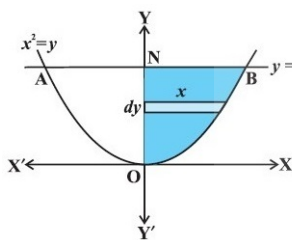


Corner Points	Corresponding value of $Z = 100x + 170y$
(0, 0)	0
(1200, 0)	$1200 \times 100 = 12000$
(1080, 180)	$100 \times 1080 + 170 \times 180 = 138600$ (maximum)
(0, 450)	$0 + 170 \times 450 = 76500$

Hence, the maximum is 138600.

Section D

32. The required area is shown in fig below by shaded region;



The points of intersection of two curves can be calculated and are (-1,1) and (2,4) as shown in fig.

The required area is given as;

$$2 \int_0^4 x dy = 2 \text{ (area of the region BONB bounded by curve, y - axis and the lines } y = 0 \text{ and } y = 4 \text{)}$$

$$= 2 \int_0^4 \sqrt{y} dy = 2 \times \frac{2}{3} \left[y^{\frac{3}{2}} \right]_0^4 = \frac{4}{3} \times 8 = \frac{32}{3}$$

which is the required area.

33. According to the question, $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$

$$\text{and } \vec{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}.$$

$$\text{Now, } \vec{b} + \vec{c} = 2\hat{i} + 4\hat{j} - 5\hat{k} + \lambda\hat{i} + 2\hat{j} + 3\hat{k}$$

$$= (2 + \lambda)\hat{i} + 6\hat{j} - 2\hat{k}$$

$$\therefore |\vec{b} + \vec{c}| = \sqrt{(2 + \lambda)^2 + (6)^2 + (-2)^2}$$

$$= \sqrt{4 + \lambda^2 + 4\lambda + 36 + 4}$$

$$= \sqrt{\lambda^2 + 4\lambda + 44}$$

The unit vector along $\vec{b} + \vec{c}$

$$= \frac{\vec{b} + \vec{c}}{|\vec{b} + \vec{c}|} = \frac{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{\lambda^2 + 4\lambda + 44}} \dots(i)$$

According to the question, the scalar product of $(\hat{i} + \hat{j} + \hat{k})$ with unit vector $\vec{b} + \vec{c}$ is 1.

$$\therefore (\hat{i} + \hat{j} + \hat{k}) \cdot \frac{\vec{b} + \vec{c}}{|\vec{b} + \vec{c}|} = 1$$

$$\Rightarrow (\hat{i} + \hat{j} + \hat{k}) \cdot \frac{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{\lambda^2 + 4\lambda + 44}} = 1$$

$$\Rightarrow \frac{1(2+\lambda) + 1(6) + 1(-2)}{\sqrt{\lambda^2 + 4\lambda + 44}} = 1$$

$$\Rightarrow \frac{(2+\lambda) + 6 - 2}{\sqrt{\lambda^2 + 4\lambda + 44}} = 1$$

$$\Rightarrow \lambda + 6 = \sqrt{\lambda^2 + 4\lambda + 44}$$

$$\Rightarrow (\lambda + 6)^2 = \lambda^2 + 4\lambda + 44$$

[squaring both sides]

$$\Rightarrow \lambda^2 + 36 + 12\lambda = \lambda^2 + 4\lambda + 44$$

$$\Rightarrow 8\lambda = 8$$

$$\Rightarrow \lambda = 1$$

The value of λ is 1.

Substituting the value of λ in Eq. (i), we get unit vector along $\vec{b} + \vec{c}$

$$\begin{aligned} &= \frac{(2+1)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{(1)^2 + 4(1) + 44}} \\ &= \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{1+4+44}} \\ &= \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{49}} = \frac{3}{7}\hat{i} + \frac{6}{7}\hat{j} - \frac{2}{7}\hat{k} \end{aligned}$$

OR

According to the question vectors are

$$\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k},$$

$$\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$$

$$\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$$

$$\text{Suppose, } \vec{p} = x\hat{i} + y\hat{j} + z\hat{k}$$

We have, $\vec{p}$ is perpendicular to both $\vec{a}$ and $\vec{b}$.

$$\vec{p} \cdot \vec{a} = 0$$

$$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} + 4\hat{j} + 2\hat{k}) = 0$$

$$\Rightarrow x + 4y + 2z = 0 \dots(i)$$

$$\text{and } \vec{p} \cdot \vec{b} = 0$$

$$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (3\hat{i} - 2\hat{j} + 7\hat{k}) = 0$$

$$\Rightarrow 3x - 2y + 7z = 0 \dots(ii)$$

Also, given $\vec{p} \cdot \vec{c} = 18$

$$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (2\hat{i} - \hat{j} + 4\hat{k}) = 0$$

$$\Rightarrow 2x - y + 4z = 18 \dots(iii)$$

Multiplying Eq. (i) by 3 and subtracting it from Eq. (ii), we get

$$-14y + z = 0$$

Multiplying Eq. (i) by 2 and subtracting it from Eq. (iii), we get

$$-9y = 18$$

$$\Rightarrow y = -2$$

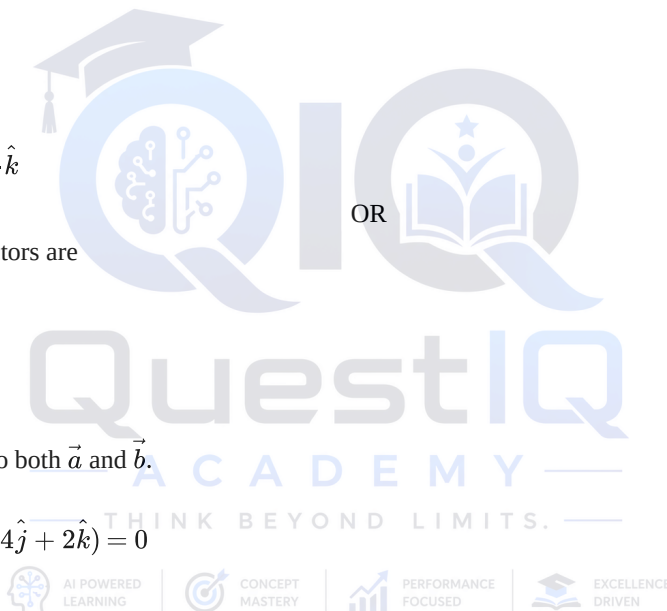
On putting $y = -2$ and $z = -28$ in Eq. (i), we get

$$x + 4(-2) + 2(-28) = 0$$

$$\Rightarrow x - 8 - 56 = 0$$

$$\Rightarrow x = 64$$

Hence, the required vector is



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$$\vec{p} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\text{i.e. } \vec{p} = 64\hat{i} - 2\hat{j} - 28\hat{k}$$

34. For $n = 1$

$$A^1 = \begin{bmatrix} 3^{1-1} & 3^{1-1} & 3^{1-1} \\ 3^{1-1} & 3^{1-1} & 3^{1-1} \\ 3^{1-1} & 3^{1-1} & 3^{1-1} \end{bmatrix} = \begin{bmatrix} 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Result is true for $n = 1$

Let it be true for $n = k$

$$A^k = \begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix}$$

Therefore $A^{k+1} = A \cdot A^k$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix}$$

$$= \begin{bmatrix} 3 \cdot 3^{k-1} & 3 \cdot 3^{k-1} & 3 \cdot 3^{k-1} \\ 3 \cdot 3^{k-1} & 3 \cdot 3^{k-1} & 3 \cdot 3^{k-1} \\ 3 \cdot 3^{k-1} & 3 \cdot 3^{k-1} & 3 \cdot 3^{k-1} \end{bmatrix}$$

$$= \begin{bmatrix} 3^k & 3^k & 3^k \\ 3^k & 3^k & 3^k \\ 3^k & 3^k & 3^k \end{bmatrix}$$

Thus, result is true for $n = k+1$

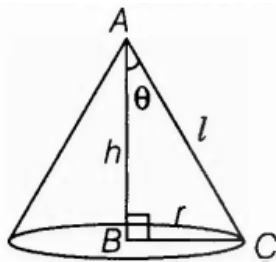
Whenever it is true for $n = k$.

Hence proved.

35. Let θ be the semi-vertical angle of the cone.

It is clear that $\theta \in (0, \frac{\pi}{2})$

Let r , h and l be the radius, height and the slant height of the cone, respectively.



Since, slant height of the cone is given, so consider it as constant.

Now, in $\triangle ABC$, $r = l \sin \theta$ and $h = l \cos \theta$

Let V be the volume of the cone.

$$\text{Then, } V = \frac{\pi}{3} r^2 h$$

$$\Rightarrow V = \frac{1}{3} \pi (l^2 \sin^2 \theta) (l \cos \theta)$$

$$\Rightarrow V = \frac{1}{3} \pi l^3 \sin^2 \theta \cos \theta$$

On differentiating both sides w.r.t. θ , we get,

$$\frac{dV}{d\theta} = \frac{l^3 \pi}{3} [\sin^2 \theta (-\sin \theta) + \cos \theta (2 \sin \theta \cos \theta)]$$

$$= \frac{l^3 \pi}{3} (-\sin^3 \theta + 2 \sin \theta \cos^2 \theta)$$

Again, differentiating both sides w.r.t θ , we get,

$$\frac{d^2 V}{d\theta^2} = \frac{l^3 \pi}{3} (-3 \sin^2 \theta \cos \theta + 2 \cos^3 \theta - 4 \sin^2 \theta \cos \theta)$$

$$\Rightarrow \frac{d^2 V}{d\theta^2} = \frac{l^3 \pi}{3} (2 \cos^3 \theta - 7 \sin^2 \theta \cos \theta)$$

For maxima or minima, put $\frac{dV}{d\theta} = 0$

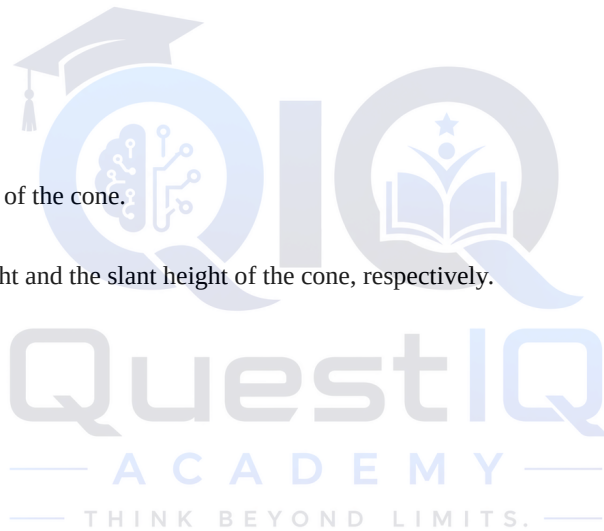
$$\Rightarrow \sin^3 \theta = 2 \sin \theta \cos^2 \theta$$

$$\Rightarrow \tan^2 \theta = 2$$

$$\Rightarrow \tan \theta = \sqrt{2} \Rightarrow \theta = \tan^{-1} \sqrt{2}$$

Now, when $\theta = \tan^{-1} \sqrt{2}$ then $\tan^2 \theta = 2$

$$\Rightarrow \sin^2 \theta = 2 \cos^2 \theta$$



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Now, we have

$$\frac{d^2V}{d\theta^2} = \frac{l^3\pi}{3}(2\cos^3\theta - 14\cos^3\theta) \\ = -4\pi l^3\cos^3\theta < 0, \text{ for } \theta \in (0, \frac{\pi}{2})$$

$\therefore V$ is maximum, when $\theta = \tan^{-1} \sqrt{2}$

$$\text{or } \theta = \cos^{-1} \frac{1}{\sqrt{3}} \left[\because \cos \theta = \frac{1}{\sqrt{1+\tan^2 \theta}} = \frac{1}{\sqrt{1+2}} = \frac{1}{\sqrt{3}} \right]$$

Hence, for given slant height, the semi-vertical angle of the cone of maximum volume is $\cos^{-1} \frac{1}{\sqrt{3}}$.

OR

Here, it is given that A right circular cylinder is inscribed inside a cone.

The curved surface area is maximum, suppose that

' r_1 ' be the radius of the cone.

' h_1 ' be the height of the cone.

' r ' be the radius of the inscribed cylinder.

' h ' be the height of the inscribed cylinder.

DF = r , and AD = AL - DL = $h_1 - h$

Now, here $\triangle ADF$ and $\triangle ALC$ are similar,

Then,

$$\frac{AD}{AL} = \frac{DF}{LC} \Rightarrow \frac{h_1 - h}{h_1} = \frac{r}{r_1}$$

$$h_1 - h = \frac{rh_1}{r_1}$$

$$h = h_1 - \frac{rh_1}{r_1} = h_1 \left(1 - \frac{r}{r_1} \right)$$

$$h = h_1 \left(1 - \frac{r}{r_1} \right) \dots (i)$$

Now let us consider the curved surface area of the cylinder,

$$S = 2\pi rh$$

Putting, h in the formula,

$$S = 2\pi r \left[h_1 \left(1 - \frac{r}{r_1} \right) \right]$$

$$S = 2\pi rh_1 - \frac{2\pi h_1 r^2}{r_1} \dots (ii)$$

For finding the maximum/ minimum of given function, we can find it by differentiating it with r and then equating it to zero. This is because if the function $S(r)$ has a maximum/minimum at a point c then $S'(c) = 0$.

Differentiating the equation (ii) with respect to r , we get

$$\frac{dS}{dr} = \frac{d}{dr} \left[2\pi rh_1 - \frac{2\pi h_1 r^2}{r_1} \right]$$

$$\frac{dS}{dr} = 2\pi h_1 - \frac{2\pi h_1 (2r)}{r_1}$$

$$\left[\text{Since } \frac{d}{dx} (x^n) = nx^{n-1} \right]$$

$$\frac{dS}{dr} = 2\pi h_1 - \frac{4\pi h_1 r}{r_1} \dots (iii)$$

To find the critical point, we need to equate equation (iii) to zero, we get

$$\frac{dS}{dr} = 2\pi h_1 - \frac{4\pi h_1 r}{r_1} = 0$$

$$\frac{4\pi h_1 r}{r_1} = 2\pi h_1$$

$$r = \frac{2\pi h_1 r_1}{4\pi h_1}$$

$$r = \frac{r_1}{2}$$

Now to check if this critical point will determine the maximum volume of the inscribed cylinder, we need to check with second differential which needs to be negative.

Consider differentiating the equation (iii) with r , we get

$$\frac{d^2S}{dr^2} = \frac{d}{dr} \left[2\pi h_1 - \frac{4\pi h_1 r}{r_1} \right]$$

$$\frac{d^2S}{dr^2} = 0 - \frac{4\pi h_1}{r_1} = -\frac{4\pi h_1}{r_1} \dots (iv)$$

$$\left[\text{Since } \frac{d}{dx} (x^n) = nx^{n-1} \right]$$

Now let us find the value of

$$\frac{d^2S}{dr^2} \text{ at } r = \frac{r_1}{2} = -\frac{4\pi h_1}{r_1}$$



As $\frac{d^2S}{dr^2}r = \frac{r_1}{2} = -\frac{4\pi h_1}{r_1} < 0$, thus, the function S is maximum at $r = \frac{r_1}{2}$ therefore,

Putting r in equation (i)

$$h = h_1 \left(1 - \frac{\frac{r_1}{2}}{r_1} \right)$$

$$h = h_1 \left(1 - \frac{1}{2} \right) = \frac{h_1}{2} \dots (v)$$

As S is maximum, from (v) we can clearly say that $h_1 = 2h$ and

$$r_1 = 2r$$

this means the radius of the cone is twice the radius of the cylinder or equal to diameter of the cylinder.

Section E

36. i. Let A be the event of committing an error and E_1, E_2 and E_3 be the events that Govind, Priyanka and Tahseen processed the form.

$$P(E_1) = 0.5, P(E_2) = 0.2, P(E_3) = 0.3$$

$$P\left(\frac{A}{E_1}\right) = 0.06, P\left(\frac{A}{E_2}\right) = 0.04, P\left(\frac{A}{E_3}\right) = 0.03$$

Using Bayes' theorem, we have

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1) \cdot P\left(\frac{A}{E_1}\right)}{P(E_1) \cdot P\left(\frac{A}{E_1}\right) + P(E_2) \cdot P\left(\frac{A}{E_2}\right) + P(E_3) \cdot P\left(\frac{A}{E_3}\right)}$$

$$= \frac{0.5 \times 0.06}{0.5 \times 0.06 + 0.2 \times 0.04 + 0.3 \times 0.03} = \frac{30}{47}$$

$$\therefore \text{Required probability} = P\left(\frac{E_1}{A}\right)$$

$$= 1 - P\left(\frac{E_1}{A}\right) = 1 - \frac{30}{47} = \frac{17}{47}$$

- ii. Let A be the event of committing an error and E_1, E_2 and E_3 be the events that Govind, Priyanka and Tahseen processed the form.

$$P(E_1) = 0.5, P(E_2) = 0.2, P(E_3) = 0.3$$

$$P\left(\frac{A}{E_1}\right) = 0.06, P\left(\frac{A}{E_2}\right) = 0.04, P\left(\frac{A}{E_3}\right) = 0.03$$

$$P(A \cap E_2) = P\left(\frac{A}{E_2}\right) \cdot P(E_2)$$

$$\Rightarrow 0.04 \times 0.2 = 0.008$$

- iii. Let A be the event of committing an error and E_1, E_2 and E_3 be the events that Govind, Priyanka and Tahseen processed the form.

$$P(E_1) = 0.5, P(E_2) = 0.2, P(E_3) = 0.3$$

$$P\left(\frac{A}{E_1}\right) = 0.06, P\left(\frac{A}{E_2}\right) = 0.04, P\left(\frac{A}{E_3}\right) = 0.03$$

$$P(A) = P\left(\frac{A}{E_1}\right) \cdot P(E_1) + P\left(\frac{A}{E_2}\right) \cdot P(E_2) + P\left(\frac{A}{E_3}\right) \cdot P(E_3)$$

$$= 0.5 \times 0.06 + 0.2 \times 0.04 + 0.3 \times 0.03 = 0.047$$

OR

Let A be the event of committing an error and E_1, E_2 and E_3 be the events that Govind, Priyanka and Tahseen processed the form.

$$P(E_1) = 0.5, P(E_2) = 0.2, P(E_3) = 0.3$$

$$P\left(\frac{A}{E_1}\right) = 0.06, P\left(\frac{A}{E_2}\right) = 0.04, P\left(\frac{A}{E_3}\right) = 0.03$$

$$\sum_{i=1}^3 P\left(\frac{E_i}{A}\right) = P\left(\frac{E_1}{A}\right) + P\left(\frac{E_2}{A}\right) + P\left(\frac{E_3}{A}\right)$$

$$= 1 \quad [\because \text{Sum of posterior probabilities is 1}]$$

37. i. If a set P has m elements and set Q has n elements then the number of functions possible from P to Q is n^m .

So, number of functions from A to B = 6^2

- ii. As the total number of Relations that can be defined from a set P to Q is the number of possible subsets of $P \times Q$.

If $n(P) = m$ and $n(Q) = n$ then $n(P \times Q) = mn$ and the number of subsets of $P \times Q = 2^{mn}$.

So number of relations possible from A to B = $2^{2 \times 6} = 2^{12}$

If $n(A) = p$ and $n(B) = q$ then $n(A \times B) = pq$ and the number of subsets of $A \times B = 2^{pq}$.

iii. $R = \{(1, 2), (2, 2), (1, 3), (3, 4), (3, 1), (4, 3), (5, 5), \}$

R is not reflexive. $(3, 3) \notin R$

R is not symmetric.

Because for $(1, 2) \in R$ there

$(2, 1) \notin R$.

R is not transitive.

Because for all element of B there does not exist,

$(a, b) (b, c) \in R$ and $(a, c) \in R$.

OR

R is reflexive, since every element of B i.e,

$B = \{1, 2, 3, 4, 5, 6\}$ is divisible by itself.

i.e, $(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) \in R$

Further, $(1, 2) \in R$

But $(2, 1) \notin R$

Moreover,

$(1, 2), (2, 4) \in R$

$\Rightarrow (1, 4) \in R$

$\Rightarrow R$ is transitive.

Therefore, R is reflexive and transitive but not symmetric.

38. i. R.H.D. of $f(x)$ at $x = 1 = \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h}$

$$= \lim_{h \rightarrow 0} \frac{|1+h-3| - |-2|}{h} = \lim_{h \rightarrow 0} \frac{2-h-2}{h} = -1$$

ii. L.H.D. of $f(x)$ at $x = 1 = \lim_{h \rightarrow 0} \frac{f(1-h) - f(1)}{-h}$

$$= \lim_{h \rightarrow 0} \frac{\left[\frac{(1-h)^2}{4} - \frac{3(1-h)}{2} + \frac{13}{4} - 2 \right]}{-h}$$

$$= \lim_{h \rightarrow 0} \left[\frac{h^2 - 2h + 1 - 6 + 6h + 13 - 8}{-4h} \right]$$

$$= \lim_{h \rightarrow 0} \left[\frac{h^2 + 4h}{-4h} \right] = -1.$$

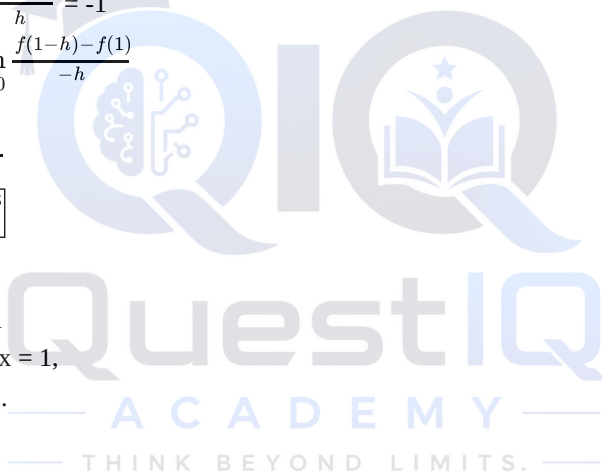
iii. Since L.H.D. of $f(x)$ at $x = 1$ is same as R.H.D. of $f(x)$ at $x = 1$, $f(x)$ is differentiable at $x = 1$.

OR

$$f(x) = \begin{cases} x - 3, & x \geq 3 \\ 3 - x, & 1 \leq x < 3 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, & x < 1 \end{cases}$$

$$[f'(x)]_{x=2} = 0 - 1 = -1$$

$$[f'(x)]_{x=-1} = \frac{2(-1)}{4} - \frac{3}{2} = -2$$



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