## Section A

1. **(a)** 5

**Explanation:**

$\because A = A' \Rightarrow \begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix} = \begin{bmatrix} 4 & 2x-3 \\ x+2 & x+1 \end{bmatrix}$

$\Rightarrow$ 2x - 3 = x + 2 $\Rightarrow$ x = 5

2. **(c)** -81

**Explanation:**

$|3AB| = 3^3|A||B| = 27(-1)(3) = -81$

3. **(a)** $\begin{vmatrix} 4/11 & 1/11 \\ -3/11 & 2/11 \end{vmatrix}$

**Explanation:**

A = $\begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix}$, adj A = $\begin{vmatrix} 4 & 1 \\ -3 & 2 \end{vmatrix}$

|A| = 8 + 3 = 11

$\therefore A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{11}\begin{vmatrix} 4 & 1 \\ -3 & 2 \end{vmatrix} = \begin{vmatrix} 4/11 & 1/11 \\ -3/11 & 2/11 \end{vmatrix}$

4. **(a)** $\tan\theta$

**Explanation:**

$x = a(\cos\theta + \theta\sin\theta)$ ,we get

$\therefore \frac{dx}{d\theta} = a(-\sin\theta + \sin\theta + \theta\cos\theta)$

$\Rightarrow \frac{d\theta}{dx} = \frac{1}{a\theta\cos\theta}$

$y = a(\sin\theta - \theta\cos\theta)$ ,we get

$\therefore \frac{dy}{d\theta} = a(\cos\theta - (\cos\theta + \theta(-\sin\theta))$

$\Rightarrow \frac{dy}{d\theta} = a\cos\theta - a\cos\theta + \theta a\sin\theta$

$\Rightarrow \frac{dy}{d\theta} = a\theta\sin\theta$

$\Rightarrow \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$

$\Rightarrow \frac{dy}{dx} = a\theta\sin\theta \times \frac{1}{a\theta\cos\theta}$

$\Rightarrow \frac{dy}{dx} = \tan\theta$

5. **(b)** $\frac{x-1}{0} = \frac{y-1}{0} = \frac{z-1}{1}$

**Explanation:**

$\frac{x-1}{0} = \frac{y-1}{0} = \frac{z-1}{1}$

6. **(d)** $y e^x + x^2 = C$

**Explanation:**

It is given that $e^x dy + (ye^x + 2x)\, dx = 0$

$\Rightarrow e^x\frac{dy}{dx} + ye^x + 2x = 0$

$\Rightarrow \frac{dy}{dx} + y = -2xe^{-x}$

This is equation in the form of $\frac{dy}{dx} + py = Q$ (where, p = 1 and Q = $-2xe^{-x}$)

Now, I.F = $e^{\int pdx} = e^{\int dx} = e^x$

Thus, the solution of the given differential equation is given by the relation:

$y(I.F.) = \int (Q \times I.F.)dx + C$

$\Rightarrow \mathrm{ye^x} = \int \left(-2\mathrm{xe^{-x}} \cdot \mathrm{e^x}\right)\mathrm{dx} + \mathrm{C}$

$\Rightarrow \mathrm{ye^x} = -\int 2\mathrm{xdx} + \mathrm{C}$

$\Rightarrow$ $ye^x = -x^2 + C$

$\Rightarrow$ $ye^x + x^2 = C$

7. **(a)** any point on the line segment joining the points (0, 2) and (3, 0).

**Explanation:**

Here the objective function is given by : F = 4x +6y.

| **Corner points** | **Corresponding value of F = 4x + 6y** |
|---|---|
| (0, 2) | 12 $\leftarrow$ Minimum |
| (3, 0) | 12 $\leftarrow$ Minimum |
| (6, 0) | 24 |
| (6, 8) | 72 |
| (0, 5) | 30 |

Hence it is clear that the minimum value occurs at any point on the line joining the points (0,2) and (3,0)

8.

**(b)** $\frac{-\pi}{10}$

**Explanation:**

$\sin^{-1}\left(\cos\frac{3\pi}{5}\right)$

$= \sin^{-1}\left(\sin\left(\frac{\pi}{2} - \frac{3\pi}{5}\right)\right)$

$= \sin^{-1}\sin\left(\frac{5\pi-6\pi}{10}\right)$

$= \sin^{-1}\sin\left(-\frac{\pi}{10}\right)$

$= -\frac{\pi}{10}\epsilon\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

9.

**(b)** $\sin^{-1}(2x - 1) + C$

**Explanation:**

The given integral is $\int \frac{dx}{\sqrt{x-x^2}} = ?$

$\left(x - x^2\right) = \frac{1}{4} - \left(x^2 - x + \frac{1}{4}\right) = \left(\frac{1}{2}\right)^2 - \left(x - \frac{1}{2}\right)^2$

$\therefore I = \int \frac{dx}{\sqrt{\left(\frac{1}{2}\right)^2 - \left(x-\frac{1}{2}\right)^2}} = \int \frac{dx}{\sqrt{\left(\frac{1}{2}\right)^2 - t^2}} = \sin^{-1}\frac{t}{(1/2)} + C$

$= \sin^{-1}2t + C = \sin^{-1}2\left(x - \frac{1}{2}\right) + C = \sin^{-1}(2x - 1) + C$

10.

**(b)** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

**Explanation:**

In the given question the given matrix is an Identity matrix, and $I^n$ = I.I.I .......... .I(n times) = I.

11.

**(b)** 1260

**Explanation:**

We have , Maximize Z = 100x + 120y , subject to constraints 2x + 3y ≤ 30, 3x + y ≤ 17, x ≥ 0, y ≥ 0.

| Corner points | Z = 100x +120 y |
|---|---|
| P( 0 , 0 ) | 0 |
| Q(3 , 8) | 1260…………………(Max.) |
| R( 0, 10 ) | 1200 |
| S(17/3 , 0 ) | 1700/3 |

Hence the maximum value is 1260

12.

**(b)** $\hat{k}$

**Explanation:**

The vector perpendicular to both the vectors $(\hat{i} - \hat{j})$

and $\hat{i} = (\hat{i} - \hat{j}) \times \hat{i}$ = $\hat{i} \times \hat{i} - \hat{j} \times \hat{i}$

= 0 + $\hat{i} \times \hat{j} = \hat{k}$

13.

**(d)** A = abc, B = $a^3 + b^3 + c^3$

**Explanation:**

$\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ = $a(bc - a^2) - b(b^2 - ac) + c(ab - c^2)$

= $abc - a^3 - b^3 + abc + abc - c^3$

= $3abc - (a^3 + b^3 + c^3)$

$\therefore \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ = $3abc - (a^3 + b^3 + c^3)$

$\therefore$ A = abc, B = $a^3 + b^3 + c^3$

14. **(a)** $\frac{1}{2}$

**Explanation:**

$\frac{1}{2}$

15. **(a)** Not defined

**Explanation:**

It is given that equation is $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$

The given differential equation is not a polynomial equation in its derivative

Therefore, its degree is not defined.

16.

**(c)** $11\hat{i} + 9\hat{j} - 2\hat{k}$

**Explanation:**

$11\hat{i} + 9\hat{j} - 2\hat{k}$

17.

**(c)** -cot x

**Explanation:**

-cot x

18. **(a)** zero

**Explanation:**

Since the lines intersect. Hence they have a common point in them. Hence the distance will be zero.

19. **(a)** Both A and R are true and R is the correct explanation of A.

**Explanation:**

We have,

x = $t^3 + 3t^2 - 6t + 18$

Velocity, $v = \frac{dx}{dt} = 3t^2 + 6t - 6$

Thus, velocity of the particle at the end of 3 seconds is

$\left(\frac{dx}{dt}\right)_{t=3} = 3(3)^2 + 6(3) - 6$

= 27 + 18 - 6 = 39 cm/s

20.

**(c)** A is true but R is false.

**Explanation:**

A is true but R is false.

Set P contains 5 elements and set Q contains 2 elements.

Let the number of elements in P be m i.e. m=5

and the number of elements in Q be n i.e. n=2.

Number of onto function = $\sum\limits_{r=0}^{n-1} {}^n c_r (-1)^r (n-r)^m$

$= \sum\limits_{r=0}^{1} {}^2 c_r (-1)^r (2-r)^5$

$= {}^2c_0 (-1)^0 (2-0)^5 + {}^2c_1 (-1)^1 (2-1)^5$

= 32+2(-1)

= 32 -2

= 30

Hence Assertion is true.

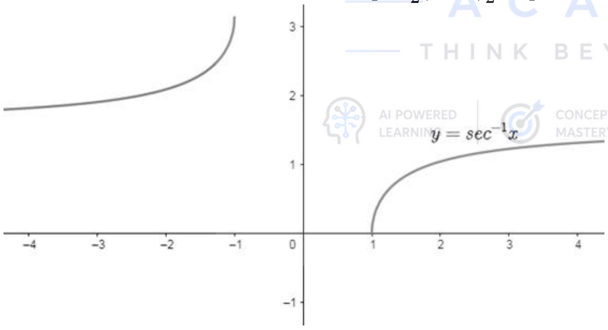
The reason is false as the number of onto functions = $\sum\limits_{r=0}^{n-1} {}^n c_r (-1)^r (n-r)^m$

so Assertion is true and reason is false.



## Section B

21. Principal value branch of sec$^{1}$ x is $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$ and its graph is shown below.




OR

We have, $\cos\left[\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) + \frac{\pi}{6}\right]$

$\cos\left[\cos^{-1}\left(-cos\frac{\pi}{6}\right) + \frac{\pi}{6}\right]$

$= \cos\left[\cos^{-1}\left(\cos\frac{5\pi}{6}\right) + \frac{\pi}{6}\right]$

$= \cos\left(\frac{5\pi}{6} + \frac{\pi}{6}\right)\{\because \cos^{-1}\cos x = x,\ x \in [0, \pi]\}$

$= \cos\left(\frac{6\pi}{6}\right)$

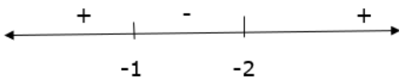
$= \cos(\pi) = -1$

22. Given,f(x) = $2x^3 + 9x^2 + 12x + 15$

f'(x) = $6x^2 + 18x + 12$

f'(x) = $6(x^2 + 3x + 2)$

= 6(x + 2)(x + 1)



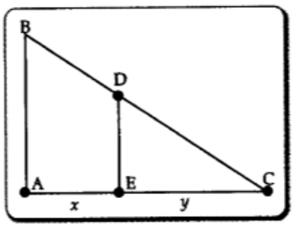

Therefore,function f(x) is decreasing for x $\in$ [-1, -2] and increasing in x $\in (-\infty,-1)\cup(-2,\infty)$

23. Let AB represent the height of the street light from the ground. At any time t seconds, let the man represented as ED of height 1.6 m be at a distance of x m from AB and the length of his shadow EC by y m.

Using similarity of triangles, we have

$\frac{4}{1.6}=\frac{x+y}{y}$

$\Rightarrow$ 3y = 2x




Differentiating both sides w.r.t. t, we get

$3\frac{dy}{dt}=2\frac{dx}{dt}$

$\frac{dy}{dt}=\frac{2}{3}\times 0.3\Rightarrow\frac{dy}{dt}=0.2$

At any time t seconds, the tip of his shadow is at a distance of (x + y) m from AB.

The rate at which the tip of his shadow moving $=\left(\frac{dx}{dt}+\frac{dy}{dt}\right)\mathrm{m/s}$ = 0.5 m/s

The rate at which his shadow is lengthening $=\frac{dy}{dt}\ \mathrm{m/s}=0.2\ \mathrm{m/s}$

OR

We have,

f(x) = $2x^3 - 3x^2 - 12x + 4$

f'(x) = $6x^2 - 6x - 12$

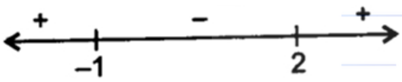
Now, f'(x) = 0

$\Rightarrow$ 6($x^2$ - x - 2) = 0

$\Rightarrow$ 6(x + 1)(x - 2) = 0

$\Rightarrow$ x = -1 and x = +2

On number line for f'(x), we get




Hence, x = -1 is point of local maxima and x = 2 is point of local minima.

24. Dividing numerator and denominator by $\cos^2 x$ we have

$I=\int\frac{\sec^2 x dx}{2\tan^2 x+5}$

Put tan x = t so that $\sec^2$xdx = dt. Then

$I=\int\frac{dt}{2t^2+5}$

$=\frac{1}{2}\int\frac{dt}{t^2+\left(\sqrt{\frac{5}{2}}\right)^2}$

$=\frac{1}{2}\left(\sqrt{\frac{2}{5}}\right)\tan^{-1}\left(\frac{t\sqrt{2}}{\sqrt{5}}\right)+C=\frac{1}{\sqrt{10}}\tan^{-1}\left(\frac{\sqrt{2}\tan x}{\sqrt{5}}\right)+C$

25. Since, we know that A(adj A) = $|A|^{n-1}$

Here, $A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$

$\therefore\ A.adjA=\begin{vmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{vmatrix}^{2-1}\times I_2$

$=1^1\times I$

$=I$

$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

But $A(\mathrm{adj}\ A)=\begin{bmatrix}k & 0\\ 0 & k\end{bmatrix}$ ...(given)

Hence, $\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow$ k = 1

Hence, k = 1

## Section C

26. We have,

$I = \int x^2 \tan^{-1} x dx$

considering $\tan^{-1}x$ as first function and $x^2$ as second function.. Then by using partial fraction we have.

$I = \tan^{-1} x \frac{x^3}{3} - \int \left(\frac{1}{1+x^2} \times \frac{x^3}{3}\right) dx$

$= \tan^{-1} x \frac{x^3}{3} - \frac{1}{3} \int \frac{x^3 dx}{1+x^2}$

$= \tan^{-1} x \frac{x^3}{3} - \frac{1}{3} \int \left(\frac{x^2 x}{1+x^2}\right) dx$

Putting $1 + x^2 = t$

$\Rightarrow x^2 = t - 1$

$\Rightarrow$ 2x dx = dt

$\Rightarrow x\mathrm{dx} = \frac{dt}{2}$

$\therefore I = \tan^{-1} x \frac{x^3}{3} - \frac{1}{6} \int \left(\frac{t-1}{t}\right) dt$

$= \frac{x^3}{3} \tan^{-1} x - \frac{1}{6} \int dt + \frac{1}{6} \int \frac{dt}{t}$

$= \frac{x^3}{3} \tan^{-1} x - \frac{1}{6} t + \frac{1}{6} \log|t| + C$

$= \frac{x^3}{3} \tan^{-1} x - \frac{1}{6}(1 + x^2) + \frac{1}{6} \log|1 + x^2| + C$

$= \frac{x^3}{3} \tan^{-1} x - \frac{x^2}{6} + \frac{1}{6} \log|x^2 + 1| + C'$ where $C' = C - \frac{1}{6}$

27. When a die is thrown, probability of getting a six $(p) = \frac{1}{6}$

Therefore, q=probability of not getting a six = 1 - p $= 1 - \frac{1}{6} = \frac{5}{6}$

i. If he gets a six in first throw, then,
   Probability of getting a six $= \frac{1}{6}$
ii. If he does not get a six, in first throw, but he gets a six in the second throw, then
   Its probability $= \frac{5}{6} \times \frac{1}{6} = \frac{5}{36}$
   Probability that he does not get a six in first two throws and he gets a six in third throw
   $= \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{25}{216}$
   Probability that he does not get a six in any of the three throws $= \left(\frac{5}{6}\right)^3 = \frac{125}{216}$
   In first throw he gets a six, will receive Re.1
   If he gets a six in second throw, he will receive Rs. (1 – 1) = 0
   If he gets a six in third throw, he will receive Rs. (– 1 – 1 + 1) = Rs. – 1
   = he will loss Rs. 1
   If he does not get a six in all three throws, he will receive Rs. (–1 –1 –1) = Rs. –3
   Expected value $= \frac{1}{6} \times 1 + \left(\frac{5}{6} \times \frac{1}{6}\right) \times 0 + \left(\frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}\right) \times (-1) + \left(\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6}\right) \times -3$
   $= \frac{1}{6} - \frac{25}{216} - \frac{375}{216} = \frac{-364}{216} = \frac{-91}{54} = Rs. \frac{91}{54}\ Loss$

28. Given integral is: $\int_1^3 \frac{dx}{x^2(x+1)}$

To Prove: $\int_1^3 \frac{dx}{x^2(x+1)} = \frac{2}{3} + \log \frac{2}{3}$

Let $I = \frac{dx}{(x^2)(x+1)}$

Using partial fraction:

Let $\frac{1}{(x^2)(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$ .....(i)

$\Rightarrow \frac{1}{(x^2)(x+1)} = \frac{A(x)(x+1)+B(x+1)+C(x^2)}{(x+1)(x^2)}$

$\Rightarrow 1 = \mathrm{A}\left(\mathrm{x}^2 + \mathrm{x}\right) + (\mathrm{Bx} + \mathrm{B}) + \mathrm{Cx}^2$

$\Rightarrow 1 = Ax^2 + Ax + B + Bx + Cx^2$

$\Rightarrow 1 = B + (A + B)x + (A + C)x^2$

Equating the coefficients of x, $x^2$ and constant value. We get:

B = 1

A + B = 0 ⇒ A = -B ⇒ A = -1

A + C = 0 ⇒ C = -A ⇒ C = 1

Put these values in equation (i)

$\Rightarrow \frac{1}{(x^2)(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$

$\Rightarrow \frac{1}{(x^2)(x+1)} = \frac{-1}{x} + \frac{1}{x^2} + \frac{1}{x+1}$

$\Rightarrow \int \frac{1}{(x^2)(x+1)} dx = \int -\frac{1}{x} dx + \int \frac{1}{(x^2)} dx + \int \frac{1}{(x+1)} dx$

$\Rightarrow \int_1^3 \frac{1}{(x^2)(x+1)} dx = \left[-\log|x| - x^{-1} + \log|x+1|\right]_1^3$

$\Rightarrow \int_1^3 \frac{1}{(x^2)(x+1)} dx = \left[-\frac{1}{x} + \log\left|\frac{x+1}{x}\right|\right]_1^3$

$= \left[-\frac{1}{3} + \log\left|\frac{3+1}{3}\right| - \left(-\frac{1}{1} + \log\left|\frac{1+1}{1}\right|\right)\right]$

$= \left[-\frac{1}{3} + \log\left|\frac{4}{3}\right| + \left(1 - \log\left|\frac{2}{1}\right|\right)\right]$

$= \left[-\frac{1}{3} + 1 + \log\left|\frac{4}{3} \times \frac{1}{2}\right|\right]$

$\Rightarrow \mathrm{I} = \left[\frac{2}{3} + \log\left|\frac{2}{3}\right]\right|$

L.H.S = R.H.S

Hence proved.

OR

Let the given integral be,

$\mathrm{I} = \int \frac{1}{\cos 2x + 3\sin^2 x} dx$

$= \int \frac{1}{(1-2\sin^2 x) + 3\sin^2 x} dx$

$= \int \frac{1}{1+\sin^2 x} dx$

Dividing numerator and denominator by $\cos^2 x$

$\Rightarrow I = \int \frac{\sec^2 x}{\sec^2 x + \tan^2 x} dx$

$= \int \frac{\sec^2 x}{1+\tan^2 x + \tan^2 x} dx$

$= \int \frac{\sec^2 x}{1+2\tan^2} dx$

$= \int \frac{\sec^2 x}{1+(\sqrt{2}\tan x)^2} dx$

Let $\sqrt{2}$ tan x = t

$\Rightarrow \sqrt{2}\sec^2 x\, dx = dt$

$\Rightarrow \sec^2 x\, dx = \frac{dt}{\sqrt{2}}$

$\therefore I = \frac{1}{\sqrt{2}} \int \frac{dt}{1+t^2}$

$= \frac{1}{\sqrt{2}} \tan^{-1}(t) + C$

$= \frac{1}{\sqrt{2}} \tan^{-1}(\sqrt{2}\tan x) + C$

29. Given that, $(x + x^3)dy = -(1 + y + x^2y)dx$

$\Rightarrow \frac{dy}{dx} = \frac{-[1+y(1+x^2)]}{x(1+x^2)}$

$\Rightarrow \frac{dy}{dx} = -\left[\frac{1}{x(1+x^2)} + \frac{y(1+x^2)}{x(1+x)^2}\right]$

$\Rightarrow \frac{dy}{dx} + \frac{y}{x} = \frac{-1}{x(1+x^2)}$

$P = \frac{1}{x}, Q = \frac{-1}{x(1+x^2)}$

$I.F. = e^{\int p\, dx} = e^{\log x} = x$

$y \times x = \int \frac{-1}{x(1+x^2)} \times x\, dx + c$

$xy = -\tan^{-1} x + c$

OR

According to the question, $\frac{dy}{dx}$ + 2ytanx = sinx

Given equation is a linear differential equation in the form $\frac{dy}{dx}$ + Py = Q

Here, P = 2tanx and Q = sinx

Now, Integration Factor(IF) = $e^{\int Pdx} = e^{\int 2\tan x dx}$

$= e^{2\int \tan x dx}$

$= e^{2\log|\sec x|}$

$= e^{\log\sec^2 x}$

$= \sec^2 x \left[\because e^{\log f(x)} = f(x)\right]$

The solution of differential equation is given by

$y \cdot (IF) = \int(\mathrm{IF})Qdx + C$

$\Rightarrow y \cdot \sec^2 x = \int \sec^2 x \cdot \sin x dx + C$

$\Rightarrow y \cdot \sec^2 x = \int \frac{\sin x}{\cos^2 x} dx + C$

$\Rightarrow y \cdot \sec^2 x = \int \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} dx + C$

$\Rightarrow y \cdot \sec^2 x = \int \tan x \cdot \sec x dx + C$

$\Rightarrow y \cdot \sec^2 x = \sec x + C$

Dividing with $\sec^2$x on both sides, we get

$\Rightarrow y = \frac{1}{\sec x} + \frac{C}{\sec^2 x}$

$\Rightarrow$y = cosx + C.$\cos^2$x

According to the question, y = 0 when x = $\frac{\pi}{3}$

$0 = \cos\frac{\pi}{3} + C \cdot \cos^2\frac{\pi}{3}$

$\Rightarrow 0 = \frac{1}{2} + C \cdot \frac{1}{4}$

$\Rightarrow \frac{-1}{2} = \frac{C}{4}$

$\Rightarrow C = -2$

Now, y = cosx + C.$\cos^2$x $\Rightarrow$ y = cos x - 2 $\cos^2$x

Therefore, the required particular solution is y = cos x - 2 $\cos^2$x

30. If $|\vec{a}|$ = 5, $|\vec{b}|$ = 12, $|\vec{c}|$ = 13

If $(\vec{a} + \vec{b} + \vec{c})$ = 0.......(i)

Find $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ = ?

Squaring the given equation (i) We get ,

$(\vec{a} + \vec{b} + \vec{c}) \cdot (\vec{a} + \vec{b} + \vec{c})$ = 0

$\vec{a} \cdot \vec{a} + \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} + \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{a} \cdot \vec{c} + \vec{c} \cdot \vec{b} + \vec{c} \cdot \vec{c}$

$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$

25 + 144 + 169 + 2(x) = 0

338 + 2x = 0

2x = -338

x = -169

hence , the required term is equal to -169.

OR

Let, $\vec{a} = 5\hat{i} - 2\hat{j} + 5\hat{k}$

Also, given $\vec{b} = 3\hat{i} + \hat{k}$

Also, let

$5\hat{i} - 2\hat{j} + 5\hat{k} = \vec{u} + \vec{v}$ ... (i)

Where **u** is parallel to **b** and **v** is perpendicular to **b.**

now **u** is parallel to **b.**

$\vec{u} = \lambda\vec{b}$

$= \lambda(3\hat{i} + \hat{k})$

$\vec{u} = 3\lambda\hat{i} + \lambda\vec{k}$.... (ii)

put value of u in equation (i)

$5\hat{i} - 2\hat{j} + 5\hat{k} = (3\lambda\hat{i} + \lambda\hat{k}) + \vec{v}$

$\vec{v} = 5\hat{j} - 2\hat{j} + 5\hat{k} - 3\lambda\hat{i} - \lambda\hat{k}$

$\vec{v} = (5 - 3\lambda)\hat{i} - 2\hat{j} + (5 - \lambda)\hat{k}$

Since, **v** is perpendicular to **b**

Then v.b =0

$[(5 - 3\lambda)\hat{i} + (-2)\hat{j} + (5 - \lambda)\hat{k}] \cdot (3\hat{i} + 0 \times \hat{j} + \tilde{k}) = 0$

(5 - 3$\lambda$)(3) + (-2)(0) + (5 - $\lambda$)(1) = 0

$15 - 9\lambda + 0 + 5 - \lambda = 0$

$20 - 10\lambda = 0$

= > -10$\lambda$ = -20

=> $\lambda$= 2

putting value of $\lambda$ in equation (ii)

$\vec{u} = 3\lambda\hat{i} + \lambda\vec{k}$

$= 3(2)\hat{i} + (2)\hat{k}$

$\vec{u} = 6\hat{i} + 2\vec{k}$

put the value of u in equation (i)

$5\hat{i} - 2\hat{j} + 5\hat{k} = \vec{u} + \vec{v}$

$5\hat{i} - 2\hat{j} + 5\hat{k} = (6\hat{i} + 2\hat{k}) + \vec{v}$

$\vec{v} = 5\hat{i} - 2\hat{j} + 5\hat{k} - 6\hat{i} - 2\hat{k}$

$\vec{v} = -\hat{i} - 2\hat{j} + 3\hat{k}$

$\vec{a} = (6\hat{i} + 2\vec{k}) + (-\hat{i} - 2\hat{j} + 3\hat{k})$

31. According to the question,if x = $\tan\left(\frac{1}{a}\log y\right)$, then we have to show that $\left(1 + x^2\right)\frac{d^2y}{dx^2} + (2x - a)\frac{dy}{dx} = 0$.

We shall use product rule of differentiation to prove the above result.

Now, x = $\tan\left(\frac{1}{a}\log y\right)$

$\Rightarrow \tan^{-1} x = \frac{1}{a}\log y$

$\Rightarrow$ a tan $^{-1}x$ = log y

On differentiating both sides w.r.t x, we get,

$a \times \frac{1}{1+x^2} = \frac{1}{y} \cdot \frac{dy}{dx}$

$\Rightarrow \quad \left(1 + x^2\right)\frac{dy}{dx} = ay$

Again, differentiating both sides w.r.t x, we get,

$\left(1 + x^2\right) \cdot \frac{d}{dx}\left(\frac{dy}{dx}\right) + \frac{dy}{dx} \cdot \frac{d}{dx}\left(1 + x^2\right) = \frac{d}{dx}(ay)$[By using product rule of derivative]

$\Rightarrow \quad \left(1 + x^2\right)\frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot (2x) = a \cdot \frac{dy}{dx}$

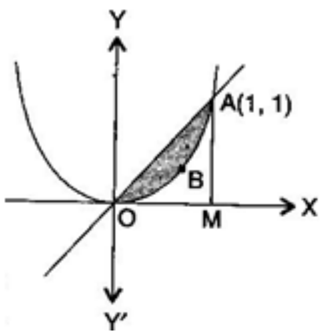
$\Rightarrow \quad \left(1 + x^2\right)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - a\frac{dy}{dx} = 0$

$\therefore \quad \left(1 + x^2\right)\frac{d^2y}{dx^2} + (2x - a)\frac{dy}{dx} = 0$

**Hence Proved.**

## Section D

32. Equation of one curve (straight line) is y = x …..(i)




Equation of second curve (parabola) is y = $x^2$ ... (ii)

Solving eq. (i) and (ii), we get x = 0 or x = 1 and y = 0 or y = 1

$\therefore$ Points of intersection of line (i) and parabola (ii) are O (0, 0) and A (1, 1).

Now Area of triangle OAM

= Area bounded by line (i) and x - axis

$= \left|\int_0^1 y dx\right| = \left|\int_0^1 x dx\right| = \left(\frac{x^2}{2}\right)_0^1$

$= \frac{1}{2} - 0 = \frac{1}{2}$ sq units

Also Area OBAM = Area bounded by parabola (ii) and x - axis

$= \left|\int_0^1 y dx\right| = \left|\int_0^1 x^2 dx\right| = \left(\frac{x^3}{3}\right)_0^1$

$= \frac{1}{3} - 0 = \frac{1}{3}$ sq. units

$\therefore$ Required area OBA between line (i) and parabola (ii)

= Area of triangle OAM – Area of OBAM

$= \frac{1}{2} - \frac{1}{3} = \frac{3-2}{6} = \frac{1}{6}$ sq. units

33. We have, $A = \{x \in Z : 0 \leq x \leq 12\}$ be a set and

R = {(a, b) : a = b} be a relation on A

Now,

Reflexivity: Let a ∈ A

⇒ a = a

⇒ (a, a) ∈ R

⇒ R is reflexive

Symmetric: Let a, b, ∈ A and (a, b) ∈ R

⇒ a = b

⇒ b = a

⇒ (b, a) ∈ R

⇒ R is symmetric

Transitive: Let a, b & c ∈ A

and let (a, b) ∈ R and (b, c) ∈ R

⇒ a = b and b = c

⇒ a = c

⇒ (a, c) ∈ R

⇒ R is transitive

Since R is being reflexive, symmetric and transitive, so R is an equivalence relation.

Also we need to find the set of all elements related to 1.

Since the relation is given by, R = {(a, b): a = b}, and 1 is an element of A.

R = {(1, 1): 1 = 1}

Thus, the set of all elements related to 1 is {1}.

OR

Given that,

R = {(1, 39), (2, 37), (3, 35) .... (19, 3), (20, 1)}

Domain = {1,2,3,.......,20}

Range = {1,3,5,7......,39}

R is not reflexive as (2, 2) ∉ R as

$2 \times 2 + 2 \neq 41$

R is not symmetric

as (1, 39) ∈ R but (39, 1) ∉ R

R is not transitive

as (11, 19) ∈ R, (19, 3) ∈ R

But (11, 3) ∉ R

Hence, R is neither reflexive, nor symmetric and nor transitive.

34. L.H.S. $I + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix} = \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix}$

Now, $I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix} = \begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix}$

R.H.S. $= (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$$= \begin{bmatrix} \cos\alpha + \sin\alpha\tan\frac{\alpha}{2} & -\sin\alpha + \cos\alpha\tan\frac{\alpha}{2} \\ -\cos\alpha\tan\frac{\alpha}{2} + \sin\alpha & \sin\alpha\tan\frac{\alpha}{2} + \cos\alpha \end{bmatrix}$$

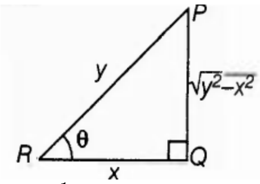
$$= \begin{bmatrix} \cos\alpha + \sin\alpha\frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} & -\sin\alpha + \cos\alpha\frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} \\ -\cos\alpha\frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} + \sin\alpha & \sin\alpha\frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} + \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\cos\alpha\cos\frac{\alpha}{2} + \sin\alpha\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} & \frac{-\sin\alpha\cos\frac{\alpha}{2} + \cos\alpha\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} \\ \frac{-\cos\alpha\sin\frac{\alpha}{2} + \sin\alpha\cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} & \frac{\sin\alpha\sin\frac{\alpha}{2} + \cos\alpha\cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\cos\left(\alpha - \frac{\alpha}{2}\right)}{\cos\frac{\alpha}{2}} & \frac{-\sin\left(\alpha - \frac{\alpha}{2}\right)}{\cos\frac{\alpha}{2}} \\ \frac{\sin\left(\alpha - \frac{\alpha}{2}\right)}{\cos\frac{\alpha}{2}} & \frac{\cos\left(\alpha - \frac{\alpha}{2}\right)}{\cos\frac{\alpha}{2}} \end{bmatrix} = \begin{bmatrix} \frac{\cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} & \frac{-\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} \\ \frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} & \frac{\cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} \end{bmatrix} = \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix}$$

$\therefore$ L.H.S. = R.H.S. Proved.

35. Let us consider a right-angled triangle with base = x and hypotenuse = y. Let x + y = k, where k is a constant. Let $\theta$ be the angle between the base and the hypotenuse. Let A be the area of the triangle, then




$A = \frac{1}{2} \times QR \times PQ = \frac{1}{2}x\sqrt{y^2 - x^2}$

$\Rightarrow \quad A^2 = \frac{x^2(y^2-x^2)}{4}$

$\Rightarrow \quad A^2 = \frac{x^2\left[(k-x)^2 - x^2\right]}{4} \; [\because y = k - x]$

$\Rightarrow \quad A^2 = \frac{k^2x^2 - 2kx^3}{4}$ .....(i)

$\Rightarrow \quad 2A\frac{dA}{dx} = \frac{2k^2x - 6kx^2}{4}$ ...(ii)

$\Rightarrow \quad \frac{dA}{dx} = \frac{k^2x - 3kx^2}{4A}$

Now, for maxima or minima

Put $\frac{dA}{dx} = 0 \Rightarrow \left(k^2x - 3kx^2\right) = 0 \Rightarrow x = \frac{k}{3}$

Differentiating (ii) w.r.t x,we get,

$2\left(\frac{dA}{dx}\right)^2 + 2A\frac{d^2A}{dx^2} = \frac{2k^2 - 12kx}{4}$ .......(iii)

Put $\frac{dA}{dx} = 0$ and $x = \frac{k}{3}$ in Eq. (iii), we get

$\frac{d^2A}{dx^2} = \frac{-k^2}{4A} < 0$

Thus, A is maximum when $x = \left(\frac{k}{3}\right)$

Now, $x = \frac{k}{3}$

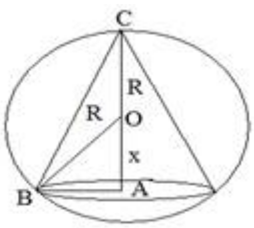
$\Rightarrow \quad y = \left(k - \frac{k}{3}\right) = \frac{2k}{3}$

$\therefore \quad \frac{x}{y} = \cos\theta$

$\Rightarrow \quad \cos\theta = \frac{k/3}{2k/3} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

Hence, the area of triangle is maximum, when $\theta = \frac{\pi}{3}$.

OR




$v = \frac{1}{3}\pi r^2 h \; \left[r^2 = \sqrt{R^2 - x^2}\right]$

$V=\frac{1}{2}\pi.\left(R^2-x^2\right).(R+x)$
$\frac{dy}{dx}=\frac{1}{3}\pi\left[\left(R^2-x^2\right)(1)+(R+x)(-2x)\right]$
$=\frac{1}{3}\pi[(R+x)(R-x)-2x(R+x)]$
$=\frac{1}{3}\pi(R+x)[R-x-2x]$
$=\frac{1}{3}\pi(R+x)(R-3x)$....(1)
Put $\frac{dv}{dr}=0$
R = - x (neglecting)
R = 3x
$\frac{R}{3}=x$
On again differentiating equation (1)
$\frac{d^2v}{dx^2}=\frac{1}{3}\pi[(R+x)(-3)+(R-3x)(1)]$
$=\left.\frac{d^2v}{dx^2}\right]_{x=\frac{R}{3}}=\frac{1}{3}\pi\left[\left(R+\frac{R}{3}\right)(-3)+\left(R-3.\frac{R}{3}\right)\right]$
$\frac{1}{3}\pi\left[\frac{4R}{3}\times-3+0\right]$
$=\frac{-1}{3}\pi4R$
$\frac{d^2v}{dx^2}<0$ Hence maximum
Now $v=\frac{1}{3}\pi\left[\left(R^2-x^2\right)(R+x)\right]\left[x=\frac{R}{3}\right]$
$v=\frac{1}{3}\pi\left[\left(R^2-\left(\frac{R}{3}\right)^2\right)\left(R+\left(\frac{R}{3}\right)\right)\right]$
$=\frac{1}{3}\pi\left[\frac{8R^2}{9}\times\frac{4R}{3}\right]$
$v=\frac{8}{27}\left(\frac{4}{3}\right)\pi R^3$
$v=\frac{8}{27}$ Volume of sphere
Volume of cone $=\frac{8}{27}$ of volume of sphere.

**Section E**

36. i. Let E denote the event that the student has failed in Economics and M denote the event that the student has failed in Mathematics.
$\therefore P(E)=\frac{50}{100}=\frac{1}{2}$ , P(M) = $\frac{35}{100}=\frac{7}{20}$ and P(E ∩ M) = $\frac{25}{100}=\frac{1}{4}$
The probability that the selected student has failed in Economics if it is known that he has failed in Mathematics.
Required probability = $P(\frac{E}{M})$
$=\frac{P(E\cap M)}{P(M)}=\frac{\frac{1}{4}}{\frac{7}{20}}=\frac{1}{4}\times\frac{20}{7}=\frac{5}{7}$

ii. Let E denote the event that student has failed in Economics and M denote the event that student has failed in Mathematics.
$\therefore P(E)=\frac{50}{100}=\frac{1}{2}$ , P(M) = $\frac{35}{100}=\frac{7}{20}$ and P(E ∩ M) = $\frac{25}{100}=\frac{1}{4}$
The probability that the selected student has failed in Mathematics if it is known that he has failed in Economics.
Required probability = P(M/E)
$=\frac{P(M\cap E)}{P(E)}=\frac{\frac{1}{4}}{\frac{1}{2}}=\frac{1}{2}$

iii. Let E denote the event that the student has failed in Economics and M denote the event that the student has failed in Mathematics.
$\therefore P(E)=\frac{50}{100}=\frac{1}{2}$ , P(M) = $\frac{35}{100}=\frac{7}{20}$ and P(E ∩ M) = $\frac{25}{100}=\frac{1}{4}$
The probability that the selected student has passed in Mathematics if it is known that he has failed in Economics
Required probability = P(M'/E)
$\Rightarrow P(M'/E)=\frac{P(M'\cap E)}{P(E)}$
$=\frac{P(E)-P(E\cap M)}{P(E)}$
$=\frac{\frac{1}{2}-\frac{1}{4}}{\frac{1}{2}}$
$\Rightarrow P(M'/E)=\frac{1}{2}$

**OR**

Let E denote the event that the student has failed in Economics and M denote the event that the student has failed in

Mathematics.

$\therefore P(E) = \frac{50}{100} = \frac{1}{2}$ , P(M) = $\frac{35}{100} = \frac{7}{20}$ and P(E ∩ M) = $\frac{25}{100} = \frac{1}{4}$

The probability that the selected student has passed in Economics if it is known that he has failed in Mathematics

Required probability = P(E'/M)

$\Rightarrow P(E'/M) = \frac{P(E' \cap M)}{P(M)}$

$= \frac{P(M) - P(E \cap M)}{P(M)}$

$= \frac{\frac{7}{20} - \frac{1}{4}}{\frac{7}{20}} \Rightarrow P(E'/M) = \frac{2}{7}$

37. i. Clearly, the coordinates of A are (8, 10, 0) and D are (0, 0, 30)

$\therefore$ Equation of AD is given by

$\frac{x-0}{8-0} = \frac{y-0}{10-0} = \frac{z-30}{-30}$

$\Rightarrow \frac{x}{4} = \frac{y}{5} = \frac{30-z}{15}$

ii. The coordinates of point C are (15, - 20, 0) and D are (0, 0, 30)

$\therefore$ Length of the cable DC

$= \sqrt{(0-15)^2 + (0+20)^2 + (30-0)^2}$

$= \sqrt{225 + 400 + 900} = \sqrt{1525} = 5\sqrt{61}$ m

iii. Since, the coordinates of point B are (-6, 4, 0) and D are (0, 0, 30), therefore vector DB is

$(-6-0)\hat{i} + (4-0)\hat{j} + (0-30)\hat{k}$ , i.e., $-6\hat{i} + 4\hat{j} - 30\hat{k}$

**OR**

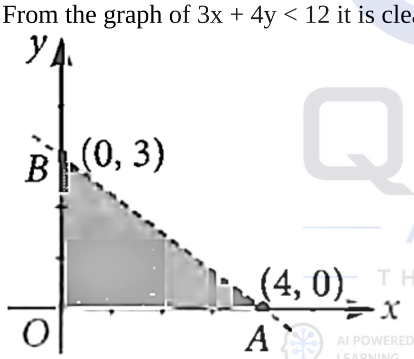
Required sum

$= (8\hat{i} + 10\hat{j} - 30\hat{k}) + (-6\hat{i} + 4\hat{j} - 30\hat{k}) + (15\hat{i} - 20\hat{j} - 30\hat{k})$

$= 17\hat{i} - 6\hat{j} - 90\hat{k}$

38. i. When we solve an L.P.P. graphically, the optimal (or optimum) value of the objective function is attained at corner points of the feasible region.

ii. From the graph of 3x + 4y < 12 it is clear that it contains the origin but not the points on the line 3x + 4y = 12.




iii. Maximum of objective function occurs at corner points.

| **Corner Points** | **Value of Z = 2x + 5y** |
|---|---|
| (0, 0) | 0 |
| (7, 0) | 14 |
| (6, 3) | 27 |
| (4, 5) | 33 ← Maximum |
| (0, 6) | 30 |

**OR**

Value of Z = px + qy at (15, 15) = 15p + 15q and that at (0, 20) = 20q. According to given condition, we have

15p + 15q = 20q ⇒ 15p = 5q ⇒ q = 3p