Section A

1.

(c) 3

Explanation:

Given, $x\begin{bmatrix}2\\3\end{bmatrix}+y\begin{bmatrix}-1\\1\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}$

$\Rightarrow \begin{bmatrix}2x\\3x\end{bmatrix}+\begin{bmatrix}-y\\y\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}\Rightarrow\begin{bmatrix}2x-y\\3x+y\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}$

$\Rightarrow$ 2x - y = 10 ...(i)

and 3x + y = 5 ...(ii)

On solving Eqs. (i) and (ii), we get x = 3

2.

(c) a^6

Explanation:

$A=\begin{bmatrix}a&0&0\\0&a&0\\0&0&a\end{bmatrix}$

$|A| = a^3$

$|\text{adj } A| = |A|^{3-1} = |A|$

$|\text{adj } A| = (a^3)^2 = a^6$

3.

(d) all integral α

Explanation:

The given system of equations has unique solution, if:

$\begin{vmatrix}1&1&1\\3&6&1\\\alpha&2&3\end{vmatrix}\neq 0$

$\Rightarrow$ 1(18 - 2) - 1(9 - α) + 1(6 - 6α) $\neq$ 0

$\Rightarrow$ 13 - 5$\alpha \neq$ 0

$\Rightarrow \alpha \neq \frac{13}{5}$. (Since α is not integral value)

Thus, unique solution exists for all integral values of α.

4.

(b) 0

Explanation:

0

5. **(a)** 4, 5, 7

Explanation:

We have,

$\frac{x-7}{2}=\frac{y+17}{-3}=\frac{z-6}{1}$

$\frac{x+5}{1}=\frac{y+3}{2}=\frac{z-4}{-2}$

The direction ratios of the given lines are proportional to 2, -3, 1 and 1, 2, -2.

The vectors parallel to the given lines are $\overrightarrow{b_1} = 2\hat{i} - 3\hat{j} + \hat{k}$ and $\overrightarrow{b_2} = \hat{i} + 2\hat{j} - 2\hat{k}$

Vector perpendicular to the vectors b_1 & b_2 is ,

$\vec{b} = \overrightarrow{b_1} \times \overrightarrow{b_2}$

= $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 1 \\ 1 & 2 & -2 \end{vmatrix}$

= $4\hat{i} + 5\hat{j} + 7\hat{k}$

Hence, the direction ratios of the line perpendicular to the given two lines are proportional to 4, 5, 7.

6.

(c) $x^2 + y^2 = Cx$

Explanation:

$(x^2 - y^2)dx + 2xydy = 0 \Rightarrow \frac{dy}{dx} = \frac{y^3 - x^3}{2xy}$

Put y = vx, we have; $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{y^2 - x^2}{2xy} \Rightarrow v + x\frac{dv}{dx} = \frac{v^2x^2 - x^2}{2xvx} \Rightarrow x\frac{dv}{dx} = -\left(\frac{v^2+1}{2v}\right)$

$\Rightarrow \int \frac{2v}{v^2+1} dv = -\int \frac{1}{x} dx \Rightarrow \log|v^2 + 1| = -\log|x| + \log C$

$\Rightarrow \log|v^2 + 1| + \log|x| = \log C \Rightarrow (v^2 + 1)|x| = C \Rightarrow (x^2 + y^2) = Cx$

7.

(b) convex polygon

Explanation:

Feasible region for an LPP is always a convex polygon.

8.

(b) $-3\hat{i} + 2\hat{j}$

Explanation:

Given that, $\alpha = \hat{k}$

and $\gamma = 2\hat{i} + 3\hat{j} + 4\hat{k}$

Since, β is perpendicular to both α and γ.

i.e., $\beta = \pm(\alpha \times \gamma) = \pm \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ 2 & 3 & 4 \end{vmatrix}$

= $\pm\hat{i}(0 - 3) - \hat{j}(0 - 2) + \hat{k}(0 - 0)$

= $\pm(-3\hat{i} + 2\hat{j})$

9. **(a)** $\frac{\pi}{2}$

Explanation:

We have,

$I = \int_0^1 \frac{d}{dx}\left\{\sin^{-1}\left(\frac{2x}{1+x^2}\right)\right\} dx$

We know since $\int f'(x) = f(x)$

$f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ and $f'(x) = \frac{d}{dx}\left\{\sin^{-1}\left(\frac{2x}{1+x^2}\right)\right\}$

Therefore, $I = \left[\sin^{-1}\left(\frac{2x}{1+x^2}\right)\right]_0^1$

= $\sin^{-1}(1) - \sin^{-1}(0)$

$= \frac{\pi}{2}$

10.

(b) $\frac{1}{3}\begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix}$

Explanation:

A + B = $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ ...(i)

A - 2B = $\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$ ...(ii)

adding 2 $\times$ (i) and (ii), we get

2A + 2B = $\begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix}$ ...(iii)

A - 2B = $\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$ ...(iv)

adding (iii) and (iv), we get

$\Rightarrow$ 3A = $\begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$

$\Rightarrow$ A = $\frac{1}{3}\begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix}$

11.

(b) (2, 3)

Explanation:

Since (2, 3) does not satisfy 2x + 3y - 12 $\leq$ 0 as 2 $\times$ 2 + 3 $\times$ 3 - 12 = 4 + 9 - 12 = 1 $\neq$ 0

12.

(b) α

Explanation:

Let $\alpha = x\hat{i} + y\hat{j} + z\hat{k}$

Then $(\alpha \cdot \hat{i})\hat{i} + (\alpha \cdot \hat{j})\hat{j} + (\alpha \cdot \hat{k})\hat{k}$

= $\{(x\hat{i} + y\hat{j} + z\hat{k}) \cdot \hat{i}\}\hat{i}$ + $\{(x\hat{i} + y\hat{j} + z\hat{k}) \cdot \hat{j}\}\hat{j}$ + $\{(x\hat{i} + y\hat{j} + z\hat{k}) \cdot \hat{k}\}\hat{k}$

= $(x)\hat{i} + (y)\hat{j} + (z)\hat{k} = \alpha$

13. **(a)** a_{11} A_{11} + a_{12} A_{12} + a_{13} A_{13}

Explanation:

By the definition of expansion of determinant, the required relation is

a_{11} A_{11} + a_{12} A_{12} + a_{13} A_{13}

14. **(a)** $\frac{7}{12}$

Explanation:

Here, $P(A) = \frac{3}{10}, P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{3}{5}$

$P(B/A) + P(A/B) = \frac{P(B \cap A)}{P(A)} + \frac{P(A \cap B)}{P(B)}$

$= \frac{P(A)+P(B)-P(A \cup B)}{P(A)} + \frac{P(A)+P(B)-P(A \cup B)}{P(B)}$

$= \frac{\frac{3}{10}+\frac{2}{5}-\frac{3}{5}}{\frac{3}{10}} + \frac{\frac{3}{10}+\frac{2}{5}-\frac{3}{5}}{\frac{2}{5}}$

$= \frac{\frac{1}{10}}{\frac{3}{10}} + \frac{\frac{1}{10}}{\frac{2}{5}} = \frac{1}{3} + \frac{1}{4} = \frac{7}{12}$

15. **(a)** $xe^{\int P_1 dy} = \int \left(Q_1 e^{\int P_1 dy}\right) dy + C$

Explanation:

The integrating factor of the given differential equation

$\frac{dx}{dy} + P_1 x = Q_1$ is $e^{\int P_1 dy}$

Thus, the general solution of the differential equation is given by,

$x(I.F.) = \int (Q_1 \times I.F.) dy + C$

$\Rightarrow x.\, e^{\int P_1 dy} = \int \left(Q_1 e^{\int P_1 dy}\right) dy + C$

16.

(d) $\hat{i}$

Explanation:
$\hat{i}$

17.

(d) No value
Explanation:
No value

18.

(c) 90^o
Explanation:
To find the angle with the z-axis, we use the relation:
$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$
where $\alpha = 30^\circ$ and $\beta = 120^\circ$. Calculating, we get:
$\cos^2 30^\circ = \frac{3}{4}, \cos^2 120^\circ = \frac{1}{4}$
Thus,
$\frac{3}{4} + \frac{1}{4} + \cos^2\gamma = 1 \implies \cos^2\gamma = 0 \implies \gamma = 90^\circ$.
So, the angle with the z-axis is 90^o.

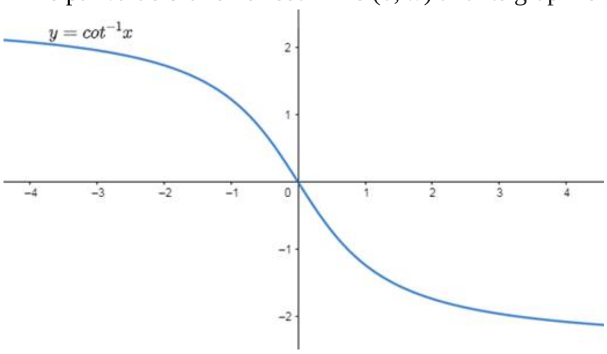
19. **(a)** Both A and R are true and R is the correct explanation of A.
Explanation:
Let y = f(x) = 15 - x^2
If x changes from 2 to 3 then δ_x = 3 - 2 = 1
Again f(3) = 15 - 9 = 16 and f(2) = 15 - 4 = 11
Therefore δ_y = f(3) - f(2) = 6 - 11 = -5

20.

(d) A is false but R is true.
Explanation:
We have, A = {1, 5, 8, 9}, B = {4, 6} and f = {(1, 4), (5, 6), (8, 4), (9, 6)}
So, all elements of B has a domain element on A or we can say elements 1 and 8 & 5 and 9 have some range 4 & 6, respectively.
Therefore, f : A $\rightarrow$ B is a surjective function not one to one function.
Also, for a bijective function, f must be both one to one onto.

Section B

21. Principal value branch of $\cot^{-1}$ x is $(0, \pi)$ and its graph is shown below.



OR

We have, $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(\frac{-\pi}{2}\right)\right]$.
$= \tan^{-1}\left(\tan\frac{5\pi}{6}\right) + \cot^{-1}\left(\cot\frac{\pi}{3}\right) + \tan^{-1}(-1)$.
$= \tan^{-1}\left[\tan\left(\pi - \frac{\pi}{6}\right)\right] + \cot^{-1}\left[\cot\left(\frac{\pi}{3}\right)\right] + tan^{-1}\left[\tan\left(\pi - \frac{\pi}{4}\right)\right]$

$= \tan^{-1}\left(-\tan\frac{\pi}{6}\right) + \cot^{-1}\left(\cot\frac{\pi}{3}\right) + \tan^{-1}\left(-\tan\frac{\pi}{4}\right)$ $\left[\begin{array}{c} \because \tan^{-1}(\tan x) = x, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \\ \cot^{-1}(\cot x) = x, x \in (0,\ \pi) \\ \text{and } \tan^{-1}(-x) = -\tan^{-1} x \end{array}\right]$

$= -\frac{\pi}{6} + \frac{\pi}{3} - \frac{\pi}{4} = \frac{-2\pi+4\pi-3\pi}{12}$

$= \frac{-5\pi+4\pi}{12} = \frac{-\pi}{12}$

22. Consider the given function,

f(x) = x^3 -15x^2 + 75x - 50

Domain of the function is R.

f'(x) = 3x^2 - 30x + 75

= 3(x^2 - 10 + 25)

= 3(x - 5) (x - 5)

= 3$(x - 5)^2$

f'(x) = 0 for x = 5

for x < 5

f'(x) > 5

and

for x > 5

f'(x) > 5

we can see throughout R the derivative is +ve but at x = 5 it is 0 so it is increasing.

23. Let r be the radius of sphere and V be its volume.

Then V = $\frac{4}{3}\pi r^3$........(i)

Given, $\frac{dV}{dt}$ = 3 cm^3/s

Differentiating (i) both sides w.r.t x,we get,

$\frac{dV}{dt} = \frac{4}{3}\pi\left(3r^2\right)\frac{dr}{dt}$

$\Rightarrow \quad 3 = \frac{4}{3}(3\pi r^2)\frac{dr}{dt}$

$\Rightarrow \quad \frac{dr}{dt} = \frac{3}{4\pi r^2}$ ....(ii)

Now, let S be the surface area of sphere, then

S = $4\pi r^2$

$\Rightarrow \quad \frac{dS}{dt} = 4\pi(2r)\frac{dr}{dt}$

$\Rightarrow \quad \frac{dS}{dt} = 8\pi r\left(\frac{3}{4\pi r^2}\right)$ [using Eq.(ii)]

$\Rightarrow \quad \left(\frac{dS}{dt}\right) = \frac{6}{r}$

when r = 2, then

$\frac{dS}{dt} = \frac{6}{2}$ = 3 cm^2/s

Therefore,the rate of inrcrease of the surface area of sphere is 3 cm^2/s when it's radius is 2 cm.

OR

Let f(x) = cos 3x

$\therefore$ f$'$(x) = -3 sin 3x

Now, $f'(x)$ = 0

$\Rightarrow$ sin 3x = 0

$\Rightarrow$ 3x = π, as $x \in \left(0, \frac{\pi}{2}\right)$

$\Rightarrow x = \frac{\pi}{3}$

The point $x = \frac{\pi}{3}$ divides the interval $\left(0, \frac{\pi}{2}\right)$ into two distinct intervals.

i.e. $\left(0, \frac{\pi}{3}\right)$ and $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

Now, in the interval, $\left(0, \frac{\pi}{3}\right)$

$f'(x) = -3\sin 3x < 0$ as $\left(0 < x < \frac{\pi}{3} \Rightarrow 0 < 3x < \pi\right)$

Therefore, 'f ' is strictly decreasing in interval $\left(0, \frac{\pi}{3}\right)$

Now, in the interval $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

$f'(x) = -3\sin 3x > 0$ as $\frac{\pi}{3} < \text{x} < \frac{\pi}{2} \Rightarrow \pi < 3\text{x} < \frac{3\pi}{2}$

Therefore, 'f ' is strictly increasing in the interval $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$.

24. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x^2 is the first function, and cos x is the second function.

Using Integration by part

$\int \text{a. b} \cdot \text{dx} = \text{a} \int \text{bdx} - \int \left[\frac{\text{da}}{\text{dx}} \cdot \int \text{bdx}\right] \text{dx}$

$\Rightarrow \int x^2 \cos x \, dx = x^2 \int \cos x \, dx - \int \left[\frac{dx^2}{dx} \cdot \int \cos x \, dx\right] dx$

$=x^2 \sin x - \int [2 x \times \sin x] dx$

$=x^2 \sin x - 2[\int x \sin x \, d x]$

Again applying by the part method in the second half, we get

$x^2 \sin x - 2 \int x \sin x \, dx$

$= x^2 \sin x - 2\left[x \int \sin x \, dx - \int \left(\frac{dx}{dx} \cdot \int \sin x \, dx\right) dx\right]$

$= x^2 \sin x - 2\left[x(-\cos x) - \int 1.(-\cos x) dx\right]$

$= x^2 \sin x - 2[-x\cos x + \sin x] + c$

$= x^2 \sin x + 2 x \cos x - 2 \sin x + c$

25. It is given that f(x) = |x + 2| - 1

Now, we can see that $|x + 2| \geq 0$ for every $x \in R$

$\Rightarrow f(x) = |x + 2| - 1 \geq -1$ for every $x \in R$

Clearly, the minimum value of f is attained when |x + 2| = 0

i.e, |x + 2| = 0

$\Rightarrow$ x = -2

Then, Minimum value of f = f(-2) = |-2 + 2| - 1 = -1

Therefore, function f does not have a maximum value.

Section C

26. According to the question, $I = \int \frac{5x-2}{1+2x+3x^2} dx$

$(5x - 2)$ can be written as ,

$5x - 2 = A\frac{d}{dx}\left(1 + 2x + 3x^2\right) + B$

$I = \int \frac{5x-2}{1+2x+3x^2} dx = \int \frac{A\frac{d}{dx}(1+2x+3x^2)+B}{1+2x+3x^2} dx$ ...(i)

$\Rightarrow 5x - 2 = A(2 + 6x) + B$

Comparing the coefficients of x and constant terms,

$5 = 6A \Rightarrow A = \frac{5}{6}$

and $-2 = 2A + B \Rightarrow B = -2A - 2$

$= -\frac{5}{3} - 2 = -\frac{11}{3} \quad \left[\because A = \frac{5}{6}\right]$

From Eq. (i), we get

$I = \int \frac{\frac{5}{6}(2+6x) - \frac{11}{3}}{1+2x+3x^2} dx$

$\Rightarrow \quad I = \int \frac{\frac{5}{6}(2+6x)}{1+2x+3x^2} dx - \int \frac{\left(\frac{11}{3}\right)}{1+2x+3x^2} dx$

$\Rightarrow I = I_1 - I_2$ ...(ii)

where, $I_1 = \frac{5}{6} \int \frac{2+6x}{1+2x+3x^2} dx$

Put $1 + 2x + 3x^2 = t \Rightarrow (2 + 6x)dx = dt$

$\therefore \quad I_1 = \frac{5}{6} \int \frac{dt}{t} = \frac{5}{6} \log|t| + C_1$

$= \frac{5}{6} \log\left|1 + 2x + 3x^2\right| + C_1 \; [t = 1 + 2x + 3x^2]$

where, $I_2 = \frac{11}{3} \int \frac{dx}{3x^2+2x+1}$

$= \frac{11}{9} \int \frac{dx}{\left[x^2+\frac{2x}{3}+\frac{1}{3}\right]}$

$= \frac{11}{9} \int \frac{dx}{\left[x^2+\frac{2x}{3}+\frac{1}{3}+\frac{1}{9}-\frac{1}{9}\right]}$

$= \frac{11}{9} \int \frac{dx}{\left[x^2+\frac{2x}{3}+\frac{1}{9}+\frac{1}{3}-\frac{1}{9}\right]}$
$= \frac{11}{9} \int \frac{dx}{\left(x+\frac{1}{3}\right)^2+\frac{2}{9}} [\because (a+b)^2 = a^2+b^2+2ab]$
$= \frac{11}{9} \cdot \frac{1}{\frac{\sqrt{2}}{3}} \tan^{-1}\left(\frac{x+\frac{1}{3}}{\frac{\sqrt{2}}{3}}\right) + C_2 \left[\because \int \frac{1}{x^2+a^2} dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c\right]$
$= \frac{11}{3\sqrt{2}} \tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right) + C_2$
Putting the values of I_1 *and* I_2 in Equation (ii),
$\Rightarrow I = \frac{5}{6}\log|1+2x+3x^2| + C_1 - \frac{11}{3\sqrt{2}}\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right) - C_2$
$\Rightarrow I = \frac{5}{6}\log|1+2x+3x^2| - \frac{11}{3\sqrt{2}}\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right) + C[\because C = C_1 - C_2]$

27. Let P(A) = Probability that A solves the problem
P(B) = Probability that B solves the problem
$P(\overline{A})$= Probability that A does not solve the problem
and $P(\overline{B})$= Probability that B does not solve the problem
According to the question, we have
$P(A) = \frac{1}{2}$
then $P(\overline{A}) = 1 - P(A) = 1 - \frac{1}{2} = \frac{1}{2}$
$[\because \quad P(A) + P(\overline{A}) = 1]$
and $P(B) = \frac{1}{3}$
then $P(\overline{B}) = 1 - P(B) = 1 - \frac{1}{3} = \frac{2}{3}$

i. P (problem is solved)
$= P(A \cap \overline{B}) + P(\overline{A} \cap B) + P(A \cap B)$
$= P(A)P(\overline{B}) + P(\overline{A}) \cdot P(B) + P(A) \cdot P(B)$
[$\because$ A and Bare independent events]
$= \left(\frac{1}{2} \times \frac{2}{3}\right) + \left(\frac{1}{2} \times \frac{1}{3}\right) + \left(\frac{1}{2} \times \frac{1}{3}\right)$
$= \frac{2}{6} + \frac{1}{6} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$
Hence, probability that the problem is solved, is $\frac{2}{3}$

ii. P (exactly one of them solve the problem)
= P (A solve but B do not solve) + P (A do not solve but B solve)
$= P(A \cap \overline{B}) + P(\overline{A} \cap B)$
$= P(A) \quad P(\overline{B}) + P(\overline{A})P(B)$
$= \left(\frac{1}{2} \times \frac{2}{3}\right) + \left(\frac{1}{2} \times \frac{1}{3}\right) = \frac{2}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$

28. We have, to write the above integral as,
$I = \int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_a^{2a} f(x)dx$
$I = \int_0^a f(x)dx + I_1$
For l_1, Let 2a - t = x then dx = -dt
t = a, x = a
t = 2a, x = 0
$I_1 = \int_0^{2a} f(x) = \int_a^0 f(2a-t)(-dt)$
$= -\int_a^0 f(2a-t)dt$
$I_1 = \int_0^a f(2a-t)dt = \int_0^a f(2a-x)dx$
$I = \int_0^a f(x)dx + \int_0^a f(2a-x)dx$
$I = \int_0^a f(x)dx - \int_0^a f(x)$ ...[$\because$ f(2a - x) = -f(x)]
I = 0
Hence,
$\int_0^{2a} f(x)dx = 0$

OR

We can write Sin 2x = 2sinx.cosx
$\int e^{\sin x} \sin 2x\, dx = 2\int e^{\sin x} .\sin x \cos x dx$

Let Sin x = t

Cos xdx = dt

$2\int e^{\sin x} \sin x \cos xdx = 2\int e^{t}\, t\, dt$

Using BY PARTS METHOD.

$2\int e^{t}\cdot t dt = 2\left[t\int e^{t}dt - \int\left(\frac{dt}{dt}\cdot\int e^{t}dt\right)dt\right]$

$= 2\left[t\cdot e^{t} - \int 1.e^{t}dt\right]$

$= 2\left[t\cdot e^{t} - e^{t}\right] + c$

$= 2e^{t}(t-1) + c$

Replacing t with sin x

$= 2e^{\sin x}(\sin x - 1) + c$

29. $x\frac{dy}{dx} + y + \frac{1}{1+x^2} = 0$

$x\frac{dy}{dx} + y = -\frac{1}{1+x^2}$

$\frac{dy}{dx} + \left(\frac{1}{x}\right)y = -\frac{1}{x(1+x^2)}$

So it a linear differential equation

P = $\frac{1}{x}$ and Q = $-\frac{1}{x(1+x^2)}$

$IF = e^{\int pdx} = e^{\int\frac{1}{x}dx} = e^{\log x} = x$

General solution

$y.IF = \int Q.IFdx + C$

$y.x = -\int\frac{1}{x(1+x^2)}.x\ dx + C$

$y.x = -\int\frac{1}{(1+x^2)}\ dx + C$

$y.x = -\tan^{-1}x + C$ ...(i)

At x =1, y = 0

$0 = -\tan^{-1}(1) + C$

$C = \frac{\pi}{4}$

Putting in eqn (i)

$y.x = -\tan^{-1}x + \frac{\pi}{4}$

OR

The given differential equation is,

$\left(1-x^2\right)\frac{dy}{dx} + xy = ax$

$\Rightarrow \frac{dy}{dx} + \frac{x}{1-x^2}\cdot y = \frac{ax}{1-x^2}$

This is of the form $\frac{dy}{dx} + Py = Q$

where $P = \frac{x}{1-x^2}$ and $Q = \frac{ax}{1-x^2}$

This the given differential equation is linear

Now, $IF = e^{\int Pdx}$

$= e^{\int\frac{xdx}{1-x^2}}$

$= e^{\frac{-1}{2}\int\frac{-2x}{1-x^2}dx} = e^{-\frac{1}{2}\log(1-x^2)} = \left(1-x^2\right)^{\frac{-1}{2}} = \frac{1}{\sqrt{1-x^2}}$

Solution is

$(IF)\cdot y = \int(IF)Q\,dx + C$

$\Rightarrow \frac{1}{\sqrt{1-x^2}}\cdot y = \int\frac{1}{\sqrt{1-x^2}}\cdot\frac{ax}{1-x^2}dx + C$

$\Rightarrow \frac{y}{\sqrt{1-x^2}} = \int\frac{ax}{(1-x^2)^{3/2}}dx + C$

$\Rightarrow \frac{y}{\sqrt{1-x^2}} = -\frac{a}{2}\int\frac{-2x}{(1-x^2)^{3/2}}dx + C$ ....(i)

Now, $I = \int\frac{-2x}{(1-x^2)^{3/2}}dx$

Let t = 1 - x^2

$\Rightarrow$ dt = -2x dx

$\therefore I = \int\frac{dt}{t^{3/2}} \Rightarrow I = \int t^{-3/2}dt$

$$\Rightarrow I=\frac{t^{\frac{-1}{2}}}{-\frac{1}{2}} \Rightarrow I=-2 \cdot \frac{1}{\sqrt{t}}=\frac{-2}{\sqrt{1-x^2}}$$
$$\therefore \frac{y}{\sqrt{1-x^2}}=\frac{-a}{2} \times \frac{-2}{\sqrt{1-x^2}}+C$$
$$\Rightarrow y=a+C\sqrt{1-x^2}$$

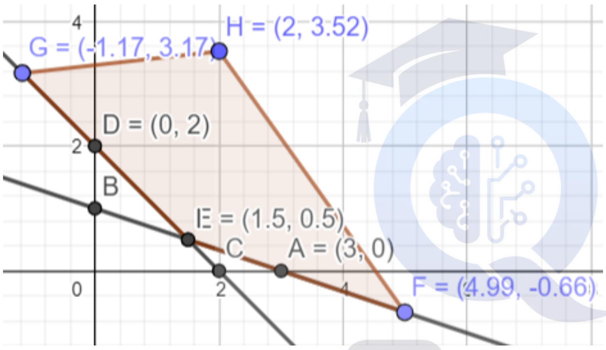
30. First, we will convert the given inequations into equations, we obtain the following equations:

$x_1 + 3x_2 = 3$, $x_1 + x_2 = 2$, $x_1 = 0$ and $x_2 = 0$

Region represented by $x_1 + 3x_2 \geq 3$:

The line $x_1 + 3x_2 = 3$ meets the coordinate axes at A(3,0) and B(0,1) respectively. By joining these points we obtain the line $x_1 + 3x_2 = 3$

Clearly (0,0) does not satisfies the inequation $x_1 + 3x_2 \geq 3$. So, the region in the plane which does not contain the origin represents the solution set of the inequation $x_1 + 3x_2 \geq 3$ Region represented by $x_1 + x_2 \geq 2$ The line $x_1 + x_2 = 2$ meets the coordinate axes at C(2,0) and D(0,2) respectively. By joining these points we obtain the line $x_1 + x_2 = 2$ Clearly (0,0) does not satisfies the inequation $x_1 + x_2 \geq 2$. So, the region containing the origin represents the solution set of the inequation $x_1 + x_2 \geq 2$ Region represented by $x_1 \geq 0$ and $x_2 \geq 0$ since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations $x_1 \geq 0$ and $x_2 \geq 0$ The feasible region determined by subject to the constraints are, $x_1 + 3x_2 \geq 3$, $x_1 + x_2 \geq 2$,and the non-negative restrictions $x_1 \geq 0$, and $x_2 \geq 0$, are as follows



The corner points of the feasible region are O(0,0), B(0,1), $E\left(\frac{3}{2}, \frac{1}{2}\right)$ and C(2,0)

The values of objective function at the corner points are as follows:

Corner point : $z=3x_1+5x_2$

O(0, 0) : $3 \times 0+5 \times 0=0$

B(0, 1) : $3 \times 0+5 \times 1=5$

$E\left(\frac{3}{2}, \frac{1}{2}\right)$: $\frac{3}{2}+5 \times \frac{1}{2}=7$

C(2, 0) : $3 \times 2+5 \times 0=6$

Therefore, the minimum value of objective function Z is 0 at the point O(0,0). Hence, $x_1 = 0$ and $x_2 = 0$ is the optimal solution of the given LPP.

Thus, the optimal value of objective function Z is 0.

OR

First, we will convert the given inequations into equations, we obtain the following equations:

4x + y = 20, 2x + 3y = 30,x = 0 and y = 0

Region represented by $4x + y \geq 20$:

The line 4x + y = 20 meets the coordinate axes at A(5,0) and B(0,20) respectively. By joining these points we obtain the line 4x + y = 20

Clearly (0,0) does not satisfies the inequation $4x + y \geq 20$.

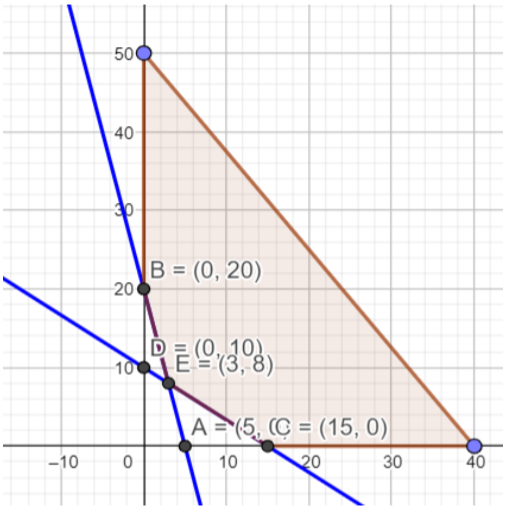
So, the region in xy plane which does not contain the origin represents the solution set of the inequation $4x + y \geq 20$

Region represented by $2x + 3y \geq 30$:

The line 2x + 3y=30 meets the coordinate axes at C(15,0) and D(0,10) respectively. By joining these points we obtain the line 2x + 3y=30 .

Clearly (0,0) does not satisfies the inequation $2x + 3y \geq 30$. So, the origin does not contain represents the solution set of the inequation $2x + 3y \geq 30$.

Region represented by $x \geq 0$ and $y \geq 0$: graph will be in first quadrant since every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the in equations x $\geq$ 0, and y $\geq$ 0 The feasible region determined by the system of constraints, 4x + y $\geq$ 20,2x + 3y $\geq$ 30,x $\geq$ 0, and y $\geq$ 0, are as follows.



The corner points of the feasible region are B(0,20), C(15,0), E(3,8) and C(15,0)

The values of Z at these corner points are as follows.

The value of objective function at the corner point : $Z = 18x + 10y$

B(0, 20) : $18 \times 0 + 10 \times 20 = 200$

E(3, 8) : $18 \times 3 + 10 \times 8 = 134$

C(15, 0) : $18 \times 15 + 10 \times 0 = 270$

Therefore, the minimum value of Z is 134 at the point E(3,8) . Hence, x = 3 and y = 8 is the optimal solution of the given LPP.

Thus, the optimal value of objective function Z is 134.

31. Given function is

f(x) = $\begin{cases} |x| + 3, & x \leq -3 \\ -2x, & -3 < x < 3 \\ 6x + 2, & x \geq 3 \end{cases}$ = $\begin{cases} -x + 3, x \leq -3 \\ -2x, -3 < x < 3 \\ 6x + 2, x \geq 3 \end{cases}$

First, we verify continuity at x = - 3 and then at x = 3

Continuity at x = -3

LHL = $\lim\limits_{x \to (-3)^-} f(x) = \lim\limits_{x \to (-3)^-} (-x + 3)$

$\Rightarrow \quad \text{LHL} = \lim\limits_{h \to 0}[-(-3 - h) + 3]$

$= \lim\limits_{h \to 0}(3 + h + 3)$

= 3 + 3 = 6

and RHL = $\lim\limits_{x \to (-3)^+} f(x) = \lim\limits_{x \to (-3)^+} (-2x)$

$\Rightarrow \quad RHL = \lim\limits_{h \to 0}[-2(-3 + h)]$

$= \lim\limits_{h \to 0}(6 - 2h)$

$\Rightarrow$ RHL = 6

Also, f(-3) = value of f(x) at x = - 3

=-(-3) + 3

= 3 + 3 = 6

$\because$ LHL = RHL = f(-3)

$\therefore$f(x) is continuous at x = - 3 So, x = - 3 is the point of continuity.

Continuity at x = 3

LHL = $\lim\limits_{x \to 3^-} f(x) = \lim\limits_{x \to 3^-} [-(2x)]$

$\Rightarrow \quad LHL = \lim\limits_{h \to 0}[-2(3 - h)]$

$= \lim\limits_{h \to 0}(-6 + 2h)$

$\Rightarrow$ LHL = -6

and RHL = $\lim\limits_{x \to 3^+} f(x) = \lim\limits_{x \to 3^+} (6x + 2)$

$\Rightarrow \quad \text{RHL} = \lim\limits_{h\to 0}[6(3+h)+2]$

$\Rightarrow$ RHL = 20

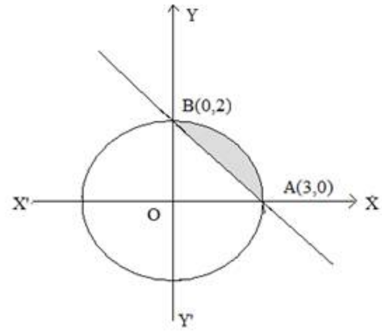
$\because \quad \text{LHL} \neq \text{RHL}$

$\therefore$ f is discontinuous at x = 3 Now, as f (x) is a polynomial function for x < -3 , -3< x < 3 and x > 3 so it is continuous in these intervals.

Hence, only x = 3 is the point of discontinuity of f(x).

Section D

32.



$\frac{x^2}{9}+\frac{y^2}{4}=1$ ....(1)

$\frac{x}{3}+\frac{y}{2}=1$ ....(2)

$\Rightarrow \frac{x^2}{(3)^2}+\frac{y^2}{(2)^2}=1$ is the equation of ellipse and $\frac{x}{3}+\frac{y}{2}=1$ is the equation of intercept form of line.

On solving (1) and (2), we get points of intersection as (0,2) and (3,0).

Area $=\frac{2}{3}\int_0^3\sqrt{9-x^2}dx-\int_0^3\left(\frac{6-2x}{3}\right)dx$

$=\frac{2}{3}[\frac{x}{2}\sqrt{3^2-x^2}+\frac{3^2}{2}sin^{-1}\frac{x}{3}]_0^3-\frac{1}{3}[6x-\frac{2x^2}{2}]_0^3$

{tex}=\frac{2}{3}[\frac{9π}{4}]-\frac{1}{3}[9]{/tex}

{tex}=\frac{3π}{2}-3{/tex}

$=\frac{3}{2}(\pi-2)$ sq unit.

33. Given that A = {1, 2, 3, 4},

a. Let R_1 = {(1,1), (1,2), (2,3), (2,2), (1,3), (3,3)}

R_1 is reflexive, since, (1,1) (2,2) (3,3) lie is R_1

Now, (1,2) $\in R_1$ (2,3) $\in R_1 \Rightarrow$ (1, 3) $\in R_1$

Hence, R_1 is also transitive but (1,2), $\in R_1 \Rightarrow$ (2,1) $\notin R_1$

So, it is not symmetric.

b. Let R_2 = {(1,2),(2,1)}. Here,1,2,3 $\in$ {1,2,3} but (1,1),(2,2),(3,3) are not in R.

Therefore, R is not reflexive.Now, (1,2) $\in R_2$, (2,1) $\in R_2$

So, it is symmetric.

Now (1,2) $\in$ R (2,1) $\in$ R, but (1,1) $\notin$ R,

therefore, R is not transitive.

c. Let R3 = {(1,2), (2,1),(1,1)(2,2),(3,3),(1,3),(3,1),(2,3)}

Clearly, R_3 is reflexive, symmetric and transitive.

OR

Given that A = [-1, 1]

i. $f(x)=\frac{x}{2}$

Let f(x_1) = f(x_2)

$\Rightarrow \frac{x_1}{2}=\frac{x_2}{2} \Rightarrow$ $x_1 = x_2$

So, f(x) is one-one.

Now, let $y=\frac{x}{2}$

$\Rightarrow x=2y\notin A,\ \forall y\in A$

As for $y=1\in A,\ x=2\notin A$

So, f(x) is not onto.

Also, f(x) is not bijective as it is not onto.

ii. g(x) = |x|

Let $g(x_1) = g(x_2)$

$\Rightarrow |x_1| = |x_2| \Rightarrow x_1 = \pm x_2$

So, g(x) is not one-one.

Now, x=±y $\notin A$ for all y ∈ R

So, g(x) is not onto, also, g(x) is not bijective.

iii. h(x) = x|x|

$\Rightarrow x_1|x_1| = x_2|x_2| \Rightarrow x_1 = x_2$

So, h(x) is one-one

Now, let y = x|x|

$\Rightarrow y = x^2 \in A, \forall x \in A$

So, h(x) is onto also, h(x) is a bijective.

iv. $k(x) = x^2$

Let $k(x_1) = k(x_2)$

$\Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = \pm x_2$

Thus, k(x) is not one-one.

Now, let $y = x^2$

$\Rightarrow x\sqrt{y} \notin A, \forall y \in A\ x = \sqrt{y} \notin A, \forall y \in A$

As for y = -1, $x = \sqrt{-1} \notin A$

Hence, k(x) is neither one-one nor onto.

34. Put $\frac{1}{x} = u$, $\frac{1}{y} = v$ and $\frac{1}{z} = w$ in the given equations,

$2u + 3v + 10w = 4;\ 4u - 6v + 5w = 1;\ 6u + 9v - 20w = 2$

$\therefore$ The matrix form of given equations is $\begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix} \begin{bmatrix} x \\ v \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$ [AX= B]

Here, $A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}, X \begin{bmatrix} x \\ v \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

$\therefore |A| = \begin{vmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{vmatrix}$

$= 2(120 - 45) - 3(-80 - 30) + 10(36 + 36)$

$= 150 + 330 + 750 = 1200 \neq 0$

$\therefore A^{-1}$exists and unique solution is $X = A^{-1}B \ldots$(i)

Now $A_{11} = 75, A_{12} = 110, A_{13} = 72$ and $A_{21} = 150, A_{22} = -100, A_{23} = 0$ and $A_{31} = 75, A_{32} = 30, A_{33} = -24$

$\therefore adj.\ A = \begin{bmatrix} 75 & 110 & 72 \\ 150 & -100 & 0 \\ 75 & 30 & -24 \end{bmatrix}' = \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$

And $A^{-1} = \frac{adj.A}{|A|} = \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$

$\therefore$ From eq. (i),

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{1200} \begin{bmatrix} 300 + 150 + 150 \\ 440 - 100 + 60 \\ 288 + 0 - 48 \end{bmatrix}$

$= \frac{1}{1200} \begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix}$

$= \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$

$\therefore u = \frac{1}{2}, v = \frac{1}{3}, w = \frac{1}{5}$

$\Rightarrow x = \frac{1}{u} = 2, y = \frac{1}{v} = 3, z = \frac{1}{w} = 5$

35. Here the equation of two planes are: $\vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 5$ and $\vec{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) = 6$.

Since the line is parallel to the two planes.

$\therefore$ Direction of line $\vec{b} = (\hat{i} - \hat{j} + 2\hat{k}) \times (3\hat{i} + \hat{j} + \hat{k})$

$= -3\hat{i} + 5\hat{j} + 4\hat{k}$

$\therefore$ Equation of required line is

$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(-3\hat{i} + 5\hat{j} + 4\hat{k})$ .... (i)

Any point on line (i) is $(1 - 3\lambda, 2 + 5\lambda, 3 + 4\lambda)$

For this line to intersect the plane $\vec{r} \cdot (2\hat{i} + \hat{j} + \hat{k})$ we have

$(1 - 3\lambda)2 + (2 + 5\lambda)1 + (3 + 4\lambda)1 = 4$

$\Rightarrow \lambda = 1$

$\therefore$ Point of intersection is (4, -3, -1)

OR

Here,it is given that

$\vec{r} = (6\hat{i} + 3\hat{k}) + \lambda(2\hat{i} - \hat{j} + 4\hat{k})$

$\vec{r} = (-9\hat{i} + \hat{j} - 10\hat{k}) + \mu(4\hat{i} + \hat{j} + 6\hat{k})$

Here,we have

$\vec{a_1} = 6\hat{i} + 3\hat{k}$

$\vec{b_1} = 2\hat{i} - \hat{j} + 4\hat{k}$

$\vec{a_2} = -9i + j - 10\hat{k}$

$\vec{b_2} = 4\hat{i} + \hat{j} + 6\hat{k}$

Thus,

$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 4 \\ 4 & 1 & 6 \end{vmatrix}$

$= i(-6 - 4) - j(12 - 16) + \hat{k}(2 + 4)$

$\therefore \vec{b_1} \times \vec{b_2} = -10\hat{i} + 4\hat{j} + 6\hat{k}$

$\therefore |\vec{b_1} \times \vec{b_2}| = \sqrt{(-10)^2 + 4^2 + 6^2}$

$= \sqrt{100 + 16 + 36}$

$= \sqrt{152}$

$\vec{a_2} - \vec{a_1} = (-9 - 6)\hat{i} + (1 - 0)\hat{j} + (6 - 3)\hat{k}$

$\therefore \vec{a_2} - \vec{a_1} = -15\hat{i} + \hat{j} + 3\hat{k}$

Now,

$(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1}) = (-10\hat{i} + 4\hat{j} + 6\hat{k}) \cdot (-15\hat{i} + \hat{j} + 3\hat{k})$

$= ((-10) \times (-15)) + (4 \times 1) + (6 \times 3)$

= 150 + 4 + 18

= 172

Thus, the shortest distance the given lines is

$d = \left| \frac{(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|} \right|$

$\therefore d = \left| \frac{172}{\sqrt{152}} \right|$

$\therefore d = \frac{172}{2\sqrt{38}}$

$\therefore d = \frac{86}{\sqrt{38}}$

$d = \frac{86}{\sqrt{38}}$ units

Section E

36. i. x + 0.21 = 0.44 $\Rightarrow$ x = 0.23

ii. 0.41 + y + 0.44 + 0.11 = 1 $\Rightarrow$ y = 0.04

iii. $P\left(\frac{C}{B}\right) = \frac{P(C \cap B)}{P(B)}$

P(B) = 0.09 + 0.04 + 0.23 = 0.36

$P\left(\frac{C}{B}\right) = \frac{0.23}{0.36} = \frac{23}{36}$

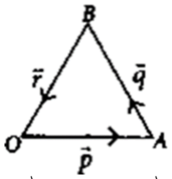
OR

P(A or B but not C)

= 0.32 + 0.09 + 0.04

= 0.45

37. i. Let OAB be a triangle such that



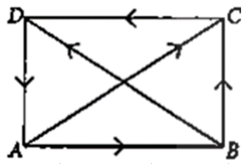
$\overrightarrow{AO} = -\vec{p}, \overrightarrow{AB} = \vec{q}, \overrightarrow{BO} = \vec{r}$

Now, $\vec{q} + \vec{r} = \overrightarrow{AB} + \overrightarrow{BO}$

$= \overrightarrow{AO} = -\vec{p}$

ii. From triangle law of vector addition,

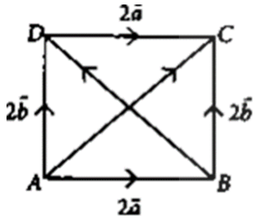
$\overrightarrow{AC} + \overrightarrow{BD} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{BC} + \overrightarrow{CD}$



= $\overrightarrow{AB} + 2\overrightarrow{BC} + \overrightarrow{CD}$

= $\overrightarrow{AB} + 2\overrightarrow{BC} - \overrightarrow{AB} = 2\overrightarrow{BC}$ $[\because \overrightarrow{AB} = -\overrightarrow{CD}]$

iii. In $\triangle$ABC, $\overrightarrow{AC}$ = $2\vec{a}$ + $2\vec{b}$ ...(i)



and in $\triangle$ABD, $2\vec{b}$ = $2\vec{a}$ + $\overrightarrow{BD}$ ...(ii) [By triangle law of addition]

Adding (i) and (ii), we have $\overrightarrow{AC} + 2\vec{b} = 4\vec{a} + \overrightarrow{BD} + 2\vec{b}$

$\Rightarrow \overrightarrow{AC} - \overrightarrow{BD} = 4\vec{a}$

OR

Since T is the mid point of YZ

So, $\overrightarrow{YT} = \overrightarrow{TZ}$

Now, $\overrightarrow{XY} + \overrightarrow{XZ} = (\overrightarrow{XT} + \overrightarrow{TY}) + (\overrightarrow{XT} + \overrightarrow{TZ})$ [By triangle law]

= $2\overrightarrow{XT} + \overrightarrow{TY} + \overrightarrow{TZ} = 2\overrightarrow{XT}$ $[\because \overrightarrow{TY} = -\overrightarrow{YT}]$

38. i. We have, I(x) = $\frac{1000}{x^2} + \frac{125}{(600-x)^2}$

$\Rightarrow$ I'(x) = $\frac{-2000}{x^3} + \frac{250}{(600-x)^3}$ and

$\Rightarrow$ I''(x) = $\frac{6000}{x^4} + \frac{750}{(600-x)^4}$

For maxima/minima, I'(x) - 0

$\Rightarrow \frac{2000}{x^3} = \frac{250}{(600-x)^3}$ $\Rightarrow$ 8(600 - x)^3 = x^3

Taking cube root on both sides, we get

2(600 - x) = x $\Rightarrow$ 1200 = 3x $\Rightarrow$ x = 400

Thus, I(x) is minimum when you are at 400 feet from the strong intensity lamp post.

ii. At a distance of 200 feet from the weaker lamp post.

Since I(x) is minimum when x = 400 feet, therefore the darkest spot between the two light is at a distance of 400 feet from a stronger lamp post, i.e., at a distance of 600 - 400 = 200 feet from the weaker lamp post.

iii. $\frac{1000}{x^2}+\frac{125}{(600-x)^2}$

Since, the distance is x feet from the stronger light, therefore the distance from the weaker light will be 600 - x.

So, the combined light intensity from both lamp posts is given by $\frac{1000}{x^2}+\frac{125}{(600-x)^2}$.

OR

We know that l(x) = $\frac{1000}{x^2}+\frac{125}{(600-x)^2}$

When x = 400

l(x) = $\frac{1000}{160000}+\frac{125}{(600-400)^2}$

= $\frac{1}{160}+\frac{125}{40000}=\frac{1}{160}+\frac{1}{320}=\frac{3}{320}$ units